Vector meson–baryon strong coupling contants in light cone QCD sum rules

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Abstract

Using the most general form of the interpolating current of the baryons, the strong coupling constants of the light vector mesons with the octet baryons are calculated within the light cone QCD sum rules. The $SU(3)_f$ symmetry breaking effects are taken into account in the calculations. It is shown that each of the electric and magnetic coupling constants can be described in terms of three universal functions. A detailed comparison of the results of this work on aforementioned couplings with the existing theoretical results is presented.

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1 Introduction

The strong coupling constants of the pseudoscalar (scalar) octet mesons π, K, η (σ, a_0, f_0), and vector nonet mesons ρ, ϕ, ω, K^* with baryons are the fundamental parameters in analysis of the existing experimental results on the meson–nucleon, nucleon–hyperon and hyperon–hyperon interactions. The coupling constants of the vector mesons with the octet baryons can be written in terms of the ρNN coupling constant and α_e (α_m), where α_e (α_m) is the $F/(F+D)$ ratio of the electric (magnetic) coupling constants [1]. The vector dominance model predicts $\alpha_e = 1$, assuming universal coupling of the ρ meson to the isospin current [2]. Therefore, reliable determination of the meson–baryon coupling constants present an important problem. Calculation of these coupling constants from the fundamental theory of strong interactions, namely QCD, represents a very important task. At the hadronic scale QCD is nonperturbative, which makes it impossible to calculate the properties of hadrons from a fundamental QCD Lagrangian. For this reason, calculation of the properties of hadrons require nonperturbative methods. Among a number of approaches, especially QCD sum rules is one of the most powerful and predictive method [3].

In this work we calculate the strong coupling constants of the octet vector mesons with baryons in the framework of the light cone QCD sum rules (LCSR) method. Note that the ρNN strong coupling constant is studied in this framework in [4]. The strong coupling constants of ρNN, $\rho\Sigma\Sigma$ and $\rho\Xi\Xi$ are studied in LCSR in [5]. The coupling constant of the vector mesons ρ and ω with the baryons is studied in the framework of the external field QCD sum rules method in [6]. The coupling constants of pseudoscalar mesons with baryons are studied comprehensively in the framework of the light cone version of the QCD sum rules [7].

Few words about the light cone QCD sum rules (LCSR) method are in order. This method is based on operator product expansion, which is carried out over twist near the light cone $x^2 \approx 0$. The matrix elements of nonlocal operators between one particle and vacuum states are parametrized in terms of distribution amplitudes, which are the main nonperturbative parameters. More about the LCSR method and its applications, can be found in [8, 9].

The paper is organized as follows. In section 2, $SU(3)_f$ classification of the vector meson–baryon coupling constants are presented, and they are calculated in LCSR framework in section 3. Section 4 is dedicated to the numerical analysis of the sum rules for the above–mentioned coupling constants and our discussions and comments on these results. In this section we also present a comparison of our results with the predictions of other approaches.

2 $SU(3)_f$ classification of the vector meson baryon coupling constants

It is well known that, in $SU(3)_f$ symmetry, coupling constants of all pseudoscalar mesons with baryons can be expressed in terms of two constants $\mathcal{F}$ and $\mathcal{D}$ in the following way:

$$\mathcal{L}_{BBM} = \sqrt{2}\mathcal{F}\,\mathrm{Tr}\bar{B}[V,B] + \sqrt{2}\mathcal{D}\,\mathrm{Tr}\bar{B}\{V,B\} - \frac{1}{\sqrt{2}}(\mathcal{F}+\mathcal{D})\mathrm{Tr}(\bar{B}B)\mathrm{Tr}V \;, \tag{1}$$

and we assume ideal mixing of the octet and singlet isosinglets giving observable ρ^0 and ω mesons. Ideal mixing corresponds to the mixing angle $\theta = \cos^{-1}\sqrt{2/3} = 35.3^0$, which is very close to the experimental value $\theta = 37.5^0$ [10]. The coefficient of the last term is chosen to eliminate the coupling of the nucleon to the pure $\bar{s}s$ state ϕ. B and V are the octet baryons and octet vector mesons:

$$B^{\alpha}_{\beta} = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{2}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix} , \quad (2)$$

$$V^{\alpha}_{\beta} = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & \rho+ & K^{*+} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix} . \quad (3)$$

Instead of the $\mathcal{F}$ and $\mathcal{D}$ as independent parameters, one can also choose to work with the coupling $g^{p\to p\rho^0}$ and the ratio $\alpha = \mathcal{F}/(\mathcal{F}+\mathcal{D})$. In terms of these parameters, $\mathcal{F} = \alpha g^{p\to p\rho^0}$, $\mathcal{D} = (1-\alpha) g^{p\to p\rho^0}$. Note also that, There are two pairs of $\mathcal{F}$ and $\mathcal{D}$ values; one for the electric type and one for the magnetic type couplings.

3 Light cone sum rules for the vector meson–baryon coupling constants

To construct LCSR for the vector meson–baryon strong coupling constants, the following correlation function is considered:

$$\Pi^{B_1\to B_2 V} = i\int d^4x e^{ipx} \left\langle V(q)\left|\mathcal{T}\left\{\eta_{B_2}(x)\bar{\eta}_{B_1}(0)\right\}\right|0\right\rangle , \quad (4)$$

where B_1 (B_2) is the initial (final) baryon, V is a vector meson, η_B is the interpolating current of the corresponding baryon, q is the momentum of V meson, and $\mathcal{T}$ is the time ordering product. The correlation function can be calculated in terms the hadrons, as well as in the deep Euclidean region $p^2 \to -\infty$, in terms of the quark and gluon degrees of freedom. Using the operator product expansion (OPE) the corresponding sum rules are obtained by equating both representations through the dispersion relations.

Let us firstly construct the phenomenological part of the correlation function. For this aim we will insert a complete set of intermediate states with the same quantum numbers as the current operators η_B. After isolating the ground state baryons, we get

$$\Pi^{B_1\to B_2 V}(p_1^2,p_2^2) = \frac{\langle 0\left|\eta_{B_2}\right| B_2(p_2)\rangle}{p_2^2-m_2^2}\langle B_2(p_2)V(q)\mid B_1(p_1)\rangle\frac{\langle B_1(p_1)\left|\bar{\eta}_{B_1}\right|0\rangle}{p_1^2-m_1^2} + \cdots , \quad (5)$$

where $p_1 = p_2 + q$, m_i is the mass of baryon B_i, and $\cdots$ represents the contributions of the higher states and the continuum.

The matrix elements entering Eq. (5) are defined as follows:

$$\begin{aligned}
\langle 0 \left| \eta_{B_i} \right| B_i(p_i) \rangle &= \lambda_{B_i} u(p_i) \ , &(6)\\
\langle B_2(p_2) V(q) \mid B_1(p_1) \rangle &= \bar{u}(p_2) \left[f_1 \gamma_\mu - f_2 \frac{i}{m_1+m_2} \sigma_{\mu\nu} q^\nu \right] u(p_1) \varepsilon^\mu \ , &(7)
\end{aligned}$$

where λ_{B_i} is the overlap amplitude for the baryon B_i, q^ν is the vector meson four–momentum, u is the Dirac spinor for the baryon which is normalized as $\bar{u}u = 2m$.

Using Eqs. (6) and (7), we obtain the following result for the phenomenological part of the correlation function:

$$\begin{aligned}
\Pi^{B_1 \to B_2} &= \frac{\lambda_{B_1} \lambda_{B_2}}{(p_1^2 - m_1^2)(p_2^2 - m_2^2)} \varepsilon^\mu (\not{p}_2 + m_2) \left\{ f_1 \gamma_\mu - f_2 \frac{i}{m_1+m_2} \sigma_{\mu\nu} q^\nu \right\} (\not{p}_1 + m_1) \ , \\
&= i \frac{\lambda_{B_1} \lambda_{B_2}}{(p^2 - m_2^2)[(p+q)^2 - m_1^2]} \Big\{ \not{p} \not{\varepsilon} \not{q} (f_1 + f_2) + 2(\varepsilon \cdot p) \not{p} f_1 + (m_1 - m_2) \not{p} \not{\varepsilon} \\
&+ 2m_2 (\varepsilon \cdot p) + (m_1 m_2 - p^2) \not{\varepsilon} f_1 + \frac{f_2}{m_1+m_2} \Big[\not{p} \not{\varepsilon} \Big((p+q)^2 - p^2 \Big) - 2(\varepsilon \cdot p) \not{p} \not{q} \\
&+ (p^2 + m_1 m_2) \not{\varepsilon} \not{q} + m_2 \Big((p+q)^2 - p^2 \Big) \not{\varepsilon} - 2m_1 (\varepsilon \cdot p) \not{q} \Big] \Big\} \\
&= \Pi^{f_1+f_2} \not{p} \not{\varepsilon} \not{q} + \Pi^{f_1} \not{p} (\varepsilon \cdot p) + \cdots \ , \qquad (8)
\end{aligned}$$

where we had set $p_1 = p$ and $p_2 = p + q$.

We see from Eq. (8) that the correlation function contains numerous structures and none of the structures has any apparent advantage over any other. Therefore any of these structures, in principle, can be used in determining the baryon–meson coupling constants. Our numerical analysis shows that the structures $\not{p}\not{\varepsilon}\not{q}$ and $\not{p}(\varepsilon \cdot p)$ exhibit better convergence, which is the reason why we choose them in further analysis. From the coefficient functions $\Pi^{f_1+f_2}$ and Π^{f_1} one can extract the values of $f_1 + f_2$ and f_1 respectively.

In order to obtain the expressions for the correlation functions, and from which the coefficient functions, from the QCD side, baryon interpolating currents are needed. In the present work we use the most general forms of the following interpolating currents for baryons:

$$\begin{aligned}
\eta^{\Sigma^0} &= \sqrt{\frac{1}{2}} \epsilon^{abc} \Big[\Big(u^{aT} C s^b \Big) \gamma_5 d^c - \Big(s^{aT} C d^b \Big) \gamma_5 u^c + \beta \Big(u^{aT} C \gamma_5 s^b \Big) d^c - \beta \Big(s^{aT} C \gamma_5 d^b \Big) u^c \Big] \ , \\
\eta^{\Sigma^+} &= -\sqrt{\frac{1}{2}} \eta^{\Sigma^0} (d \to u) \ , \\
\eta^{\Sigma^-} &= -\sqrt{\frac{1}{2}} \eta^{\Sigma^0} (u \to d) \ , \\
\eta^{p} &= -\eta^{\Sigma^+} (s \to d) \ , \\
\eta^{n} &= -\eta^{\Sigma^-} (s \to u) \ , \\
\eta^{\Xi^0} &= -\eta^{n} (d \to s) \ , \\
\eta^{\Xi^-} &= -\eta^{p} (u \to s) \ ,
\end{aligned}$$

$$
\begin{aligned}
\eta^{\Lambda} = & -\sqrt{\frac{1}{6}}\epsilon^{abc}\Big[2\Big(u^{aT}Cd^b\Big)\gamma_5 s^c + \Big(u^{aT}Cs^b\Big)\gamma_5 d^c + \Big(s^{aT}C\gamma_5 d^b\Big)u^c + 2\beta\Big(u^{aT}C\gamma_5 d^b\Big)s^c \\
& + \beta\Big(u^{aT}C\gamma_5 s^b\Big)d^c + \Big(s^{aT}C\gamma_5 d^b\Big)u^c\Big] ,
\end{aligned} \tag{9}
$$

where C is the charge conjugation operator, (a, b, c) are the color indices and β is an arbitrary parameter and $\beta = -1$ corresponds to the Ioffe current. We see from Eq. (9) that all currents, except the current of Λ, can be derived from the Σ^0 current by making simple replacements. It is shown in [11] that Λ current can also be obtained from Σ^0 current with the help of following relations:

$$
\begin{aligned}
2\eta^{\Sigma^0}(d \to s) + \eta^{\Sigma^0} &= \sqrt{3}\Lambda , \\
2\eta^{\Sigma^0}(u \to s) - \eta^{\Sigma^0} &= -\sqrt{3}\Lambda .
\end{aligned} \tag{10}
$$

Before giving detailed calculations of the correlation functions, let us firstly derive the relations among them. For this purpose we will follow the approach presented in [7], where relations between correlation functions involving coupling constants of pseudoscalar mesons to octet baryons are obtained. Of course, in the exact $SU(3)_f$ limit all coupling constants of vector mesons with octet baryons can be related to each other using symmetry arguments. The main advantage of our approach is that our approach allows us to take $SU(3)_f$ symmetry violating effects into account.

Below we will show that all correlation functions responsible for the coupling constants of the vector mesons to octet baryons can be written in terms of only three functions for each electric and magnetic form factors. Note that the relations between the invariant functions are all structure independent. Starting from the correlation function that is responsible for the $\Sigma^0 \to \Sigma^0\rho^0$ transition, two of the three independent functions can be obtained. It allows us to establish relations among this correlation function and the correlation functions responsible for $\Sigma^+ \to \Sigma^+\rho^0$ and $\Sigma^- \to \Sigma^-\rho^0$ transitions, which can be written as:

$$
\Pi^{\Sigma^0 \to \Sigma^0\rho^0} = g_{\rho\bar{u}u}\Pi_1(u, d, s) + g_{\rho\bar{d}d}\Pi_1'(u, d, s) + g_{\rho\bar{s}s}\Pi_2(u, d, s) , \tag{11}
$$

where we formally write down the quark content of the ρ^0 meson in the form

$$
J_\mu = \sum_{u,d,s} g_{\rho\bar{q}q}\bar{q}\gamma_\mu q ,
$$

and for the ρ^0 meson $g_{\rho\bar{u}u} = -g_{\rho\bar{d}d} = 1/\sqrt{2}$, $g_{\rho\bar{s}s} = 0$. The invariant functions Π_1, Π_1' and Π_2 describe emission of the ρ^0 meson from u, d and s quarks of Σ^0, respectively. We see from Eq. (9) that the current of Σ^0 is symmetric under the replacement $u \leftrightarrow d$, hence $\Pi_1'(u, d, s) = \Pi_1(d, u, s)$. For this reason, we have two independent functions $\Pi_1(u, d, s)$ and $\Pi_2(u, d, s)$. In further discussion, we introduce the following formal notation,

$$
\begin{aligned}
\Pi_1(u, d, s) &= \left\langle \bar{u}u \left| \Sigma^0\bar{\Sigma}^0 \right| 0 \right\rangle , \\
\Pi_2(u, d, s) &= \left\langle \bar{s}s \left| \Sigma^0\bar{\Sigma}^0 \right| 0 \right\rangle ,
\end{aligned} \tag{12}
$$

for convenience. Replacing $d \to u$ in $\Pi_1(d, u, s)$ and using $\Sigma^0(d \to u) = -\sqrt{2}\Sigma^+$, we obtain

$$
4\Pi_1(u, u, s) = 2\left\langle \bar{u}u \left| \Sigma^+\bar{\Sigma}^+ \right| 0 \right\rangle . \tag{13}
$$

Appearance of the factor 4 in Eq. (13) can be explained as follows. Since there are two u quarks, in Σ^+, there are two ways of contracting them. Each one of the quark lines can emit the ρ^0 meson yielding, in total, 4 ways for emitting the ρ^0 meson. Using the fact that in Σ^+ there is no d quark, we obtain:

$$\begin{aligned}\Pi^{\Sigma^+\to\Sigma^+\rho^0} &= g_{\rho\bar{u}u}\left\langle \bar{u}u\left|\Sigma^+\bar{\Sigma}^+\right|0\right\rangle + g_{\rho\bar{s}s}\left\langle \bar{s}s\left|\Sigma^+\bar{\Sigma}^+\right|0\right\rangle \\ &= \sqrt{2}\Pi_1(u,u,s)\ .\end{aligned} \tag{14}$$

Using similar arguments, for the $\Sigma^-\to\Sigma^-\rho^0$ transition, we get

$$\begin{aligned}\Pi^{\Sigma^-\to\Sigma^-\rho^0} &= g_{\rho\bar{d}d}\left\langle \bar{d}d\left|\Sigma^-\bar{\Sigma}^-\right|0\right\rangle + g_{\rho\bar{s}s}\left\langle \bar{s}s\left|\Sigma^-\bar{\Sigma}^-\right|0\right\rangle \\ &= -\sqrt{2}\Pi_1'(d,d,s) = -\sqrt{2}\Pi_1(d,d,s)\ .\end{aligned} \tag{15}$$

These equations establish relations between the couplings of ρ^0 meson with Σ^+, Σ^0 and Σ^- baryons. Note that in the isospin symmetry limit, we obtain the well known relations $\Pi^{\Sigma^+\to\Sigma^+\rho^0} = -\Pi^{\Sigma^-\to\Sigma^-\rho^0}$, and $\Pi^{\Sigma^0\to\Sigma^0\rho^0} = 0$.

Let us proceed now by calculating the couplings of ρ^0 meson with proton and neutron. For this purpose we need the matrix elements $\left\langle \bar{u}u\left|\bar{N}N\right|0\right\rangle$ and $\left\langle \bar{d}d\left|\bar{N}N\right|0\right\rangle$. The matrix involving interpolating current of the proton can be obtained from the current of Σ^+ by the replacement $s\to d$, i.e.,

$$\langle \bar{u}u\left|p\bar{p}\right|0\rangle = \left\langle \bar{u}u\left|\Sigma^+\bar{\Sigma}^+\right|0\right\rangle (s\to d) = 2\Pi_1(u,u,d)\ . \tag{16}$$

In order to obtain $\left\langle \bar{d}d\left|p\bar{p}\right|0\right\rangle$, $\Pi_2(u,d,s)$ is needed. In the first step, making the replacement $d\to u$, we get

$$\Pi_2(u,u,s) = \left\langle \bar{s}s\left|\Sigma^+\Sigma^-\right|0\right\rangle\ , \tag{17}$$

where the factor 2 in the normalization of the current is canceled by the two possible ways of contracting the u quarks. Making the replacement $s\to d$ in the second step, we obtain

$$\Pi_2(u,u,d) = \left\langle \bar{d}d\left|p\bar{p}\right|0\right\rangle\ . \tag{18}$$

It follows from Eqs. (16)–(18) that

$$\begin{aligned}\Pi^{p\to p\rho^0} &= g_{\rho\bar{u}u}\left\langle \bar{u}u\left|p\bar{p}\right|0\right\rangle + g_{\rho\bar{d}d}\left\langle \bar{d}d\left|p\bar{p}\right|0\right\rangle \\ &= \sqrt{2}\Pi_1(u,u,d) - \frac{1}{\sqrt{2}}\Pi_2(u,u,d)\ .\end{aligned} \tag{19}$$

Similarly, we can easily obtain the following results involving the coupling constants of ρ^0 meson to the neutron and Ξ baryons,

$$\begin{aligned}\Pi^{n\to n\rho^0} &= \frac{1}{\sqrt{2}}\Pi_2(d,d,u) - \sqrt{2}\Pi_1(d,d,u)\ , \\ \Pi^{\Xi^0\to\Xi^0\rho^0} &= \frac{1}{\sqrt{2}}\Pi_2(s,s,u)\ , \\ \Pi^{\Xi^-\to\Xi^-\rho^0} &= -\frac{1}{\sqrt{2}}\Pi_2(s,s,d)\ .\end{aligned} \tag{20}$$

These relations, together with the relations given in Eqs. (14), (15) and (19), describe the couplings of ρ^0 meson with baryons in terms of two invariant functions $\Pi_1(u,d,s)$ and $\Pi_2(u,d,s)$.

Now let us derive similar relations for the charged ρ meson. Consider the matrix element $\left\langle \bar{d}d \left| \Sigma^0 \bar{\Sigma}^0 \right| 0 \right\rangle$ in which d quarks from the Σ^0 and $\bar{\Sigma}^0$ form the final $\bar{d}d$ and the other u and s quarks are the spectators. In the matrix element $\left\langle \bar{u}d \left| \Sigma^+ \bar{\Sigma}^0 \right| 0 \right\rangle$, d quark from Σ^0 and u quark from Σ^+ form the state $\bar{u}d$ and the other u and s quarks, similar to the previous case, are the spectators. Therefore, one can expect that these matrix elements should be proportional, and indeed, calculations confirm that, i.e.,

$$\begin{aligned}
\Pi^{\Sigma^0 \to \Sigma^+ \rho^-} &= \left\langle \bar{u}d \left| \Sigma^+ \bar{\Sigma}^0 \right| 0 \right\rangle = -\sqrt{2} \left\langle \bar{d}d \left| \Sigma^0 \bar{\Sigma}^0 \right| 0 \right\rangle = -\sqrt{2}\Pi_1'(u,d,s) \\
&= -\sqrt{2}\Pi_1(d,u,s) \ .
\end{aligned} \tag{21}$$

Making the exchange $u \leftrightarrow d$ in Eq. (21), we get

$$\Pi^{\Sigma^0 \to \Sigma^- \rho^+} = \left\langle \bar{d}u \left| \Sigma^- \bar{\Sigma}^0 \right| 0 \right\rangle = \sqrt{2} \left\langle \bar{u}u \left| \Sigma^0 \bar{\Sigma}^0 \right| 0 \right\rangle = \sqrt{2}\Pi_1(u,d,s) \ . \tag{22}$$

Performing similar calculations for the Ξ baryons, we obtain:

$$\begin{aligned}
\Pi^{\Xi^0 \to \Xi^- \rho^+} &= \left\langle \bar{d}u \left| \Xi^- \bar{\Xi}^0 \right| 0 \right\rangle = -\sqrt{2} \left\langle \bar{u}u \left| \Xi^0 \bar{\Xi}^0 \right| 0 \right\rangle = \Pi_2(s,s,u) \ , \\
\Pi^{\Xi^- \to \Xi^0 \rho^-} &= \left\langle \bar{u}d \left| \Xi^0 \bar{\Xi}^- \right| 0 \right\rangle = -\sqrt{2} \left\langle \bar{d}d \left| \Xi^0 \bar{\Xi}^0 \right| 0 \right\rangle = \Pi_2(s,s,d) \ .
\end{aligned} \tag{23}$$

The correlation functions involving the ρ and K^* mesons can be written as:

$$\begin{aligned}
\Pi^{\Sigma^- \to \Sigma^0 \rho^-} &= \sqrt{2}\Pi_1(u,d,s) \ , \\
\Pi^{\Sigma^+ \to \Sigma^0 \rho^+} &= \sqrt{2}\Pi_1'(u,d,s) = -\sqrt{2}\Pi_1(d,u,s) \ , \\
\Pi^{\Sigma^- \to n K^{*-}} &= -\Pi_2(d,d,s) \ , \\
\Pi^{p \to \Sigma^+ K^{*0}} &= -\Pi_2(u,u,d) \ , \\
\Pi^{\Sigma^+ \to p \bar{K}^{*0}} &= -\Pi_2(u,u,s) \ , \\
\Pi^{n \to \Sigma^- K^{*+}} &= -\Pi_2(d,d,s) \ .
\end{aligned} \tag{24}$$

We make use of Eqs. (10) in order to calculate the correlation functions involving the Λ baryon, in terms of the invariant functions, after which we get:

$$\begin{aligned}
\Pi^{\Lambda \to \Lambda \rho^0} &= \frac{\sqrt{2}}{3}\Big[\Pi_1(u,s,d) - \Pi_1(d,s,u) + \Pi_2(s,d,u) - \Pi_2(s,u,d) \\
&\quad - \frac{1}{2}\Pi_1(u,d,s) + \frac{1}{2}\Pi_1(d,u,s)\Big] \ , \\
\Pi^{\Lambda \to \Sigma^0 \rho^0} + \Pi^{\Sigma^0 \to \Lambda \rho^0} &= \frac{2}{\sqrt{6}}\Big[\Pi_1(u,s,d) + \Pi_1(d,s,u) - \Pi_2(s,d,u) - \Pi_2(s,u,d)\Big] \ , \\
\Pi^{\Xi^- \to \Sigma^0 K^{*-}} + \sqrt{3}\Pi^{\Xi^- \to \Lambda K^{*-}} &= -2\sqrt{2}\Pi_1(u,s,d) \ , \\
\Pi^{n \to \Sigma^0 K^{*0}} - \sqrt{3}\Pi^{n \to \Lambda K^{*0}} &= 2\sqrt{2}\Pi_1(s,d,u) \ , \\
\Pi^{p \to \Sigma^0 K^{*+}} + \sqrt{3}\Pi^{p \to \Lambda K^{*+}} &= -2\sqrt{2}\Pi_1(s,u,d) \ , \\
-\Pi^{\Xi^0 \to \Sigma^0 K^{*0}} + \sqrt{3}\Pi^{\Xi^0 \to \Lambda K^{*0}} &= 2\sqrt{2}\Pi_1(d,s,u) \ ,
\end{aligned}$$

$$
\begin{aligned}
\Pi^{\Sigma^0\to pK^{*-}} + \sqrt{3}\Pi^{\Lambda\to pK^{*-}} &= -2\sqrt{2}\Pi_1(s,u,d) \ , \\
\Pi^{\Sigma^0\to nK^{*0}} - \sqrt{3}\Pi^{\Lambda\to nK^{*0}} &= 2\sqrt{2}\Pi_1(s,d,u) \ , \\
\Pi^{\Sigma^0\to \Xi^0\bar{K}^{*0}} - \sqrt{3}\Pi^{\Lambda\to \Xi^0\bar{K}^{*0}} &= -2\sqrt{2}\Pi_1(d,s,u) \ , \\
\Pi^{\Sigma^0\to \Xi^- K^{*+}} + \sqrt{3}\Pi^{\Lambda\to \Xi^- K^{*+}} &= 2\sqrt{2}\Pi_1(u,s,d) \ .
\end{aligned} \tag{25}
$$

As can easily be seen in Eq. (25), correlation functions involving a single Λ baryon always come together with correlation functions involving a Σ^0 baryon, and therefore it is impossible to separate them using only Π_1 and Π_2. To be able to separate the correlation functions involving the Λ and Σ^0 baryons, we need to introduce one more independent function

$$
\Pi_3(u,d,s) = -\Pi^{\Sigma^0\to\Xi^- K^{*+}} = -\left\langle u\bar{s}\left|\Xi^-\bar{\Sigma}^0\right|0\right\rangle \ . \tag{26}
$$

Note that in [7], a fourth function is defined as

$$
\Pi_4(u,d,s) = -\Pi^{\Xi^-\to\Sigma^0 K^{*-}} = -\left\langle s\bar{u}\left|\Sigma^0\bar{\Xi}^-\right|0\right\rangle \ . \tag{27}
$$

In the present work a new relation missed in [7] is obtained, namely, $\Pi_4(u,d,s) = \Pi_3(u,s,d)$, and hence such a fourth function is not necessary. Choice of $\Pi_3(u,d,s)$ is not unique.

Using this new invariant function and performing simple calculations, we obtain:

$$
\begin{aligned}
\Pi^{\Xi^0\to\Sigma^+K^{*-}} &= \Pi^{n\to p\rho^-}(d\to s) = -\sqrt{2}\Pi_3(s,s,u) \ , \\
\Pi^{\Xi^-\to\Sigma^-K^{*0}} &= \Pi^{\Xi^0\to\Sigma^+K^{*-}}(u\to d) = -\sqrt{2}\Pi_3(s,s,d) \ , \\
\Pi^{\Sigma^+\to\Xi^0K^{*+}} &= \sqrt{2}\Pi^{\Sigma^0\to\Xi^-K^{*+}}(d\to u) = -\sqrt{2}\Pi_3(u,u,s) \ , \\
\Pi^{p\to n\rho^+} &= \Pi^{\Sigma^+\to\Xi^0K^{*+}}(s\to d) = -\sqrt{2}\Pi_3(u,u,d) \ , \\
\Pi^{n\to p\rho^-} &= \Pi^{p\to n\rho^+}(u\leftrightarrow d) = -\sqrt{2}\Pi_3(d,d,u) \ , \\
\Pi^{\Sigma^0\to pK^{*-}} - \sqrt{3}\Pi^{\Lambda\to pK^{*-}} &= 2\Pi^{\Sigma^0\to\Xi^-K^{*+}}(s\leftrightarrow u) = -2\Pi_3(s,d,u) \ , \\
\Pi^{\Sigma^0\to nK^{*0}} + \sqrt{3}\Pi^{\Lambda\to nK^{*0}} &= -2\Pi^{\Sigma^0\to\Xi^-K^{*+}}(u\leftrightarrow s)(d\leftrightarrow u) = 2\Pi_3(s,u,d) \ , \\
\Pi^{\Sigma^0\to\Sigma^-\rho^+} + \sqrt{3}\Pi^{\Lambda\to\Sigma^-\rho^+} &= 2\Pi^{\Sigma^0\to\Xi^-K^{*+}}(s\leftrightarrow d) = -2\Pi_3(u,s,d) \ , \\
\Pi^{\Lambda\to\Sigma^+\rho^-} &= \Pi^{\Lambda\to\Sigma^-\rho^+}(u\to d) \ , \\
\Pi^{\Sigma^-\to\Xi^-\bar{K}^{*0}} &= \Pi^{\Sigma^+\to\Xi^0K^{*+}}(u\to d) = -\sqrt{2}\Pi_3(d,d,s) \ , \\
\Pi^{\Sigma^0\to\Xi^0\bar{K}^{*0}} &= -\Pi^{\Sigma^0\to\Xi^-K^{*+}}(d\leftrightarrow u) = \Pi_3(d,u,s) \ , \\
\Pi^{\Xi^0\to\Sigma^0K^{*0}} &= -\Pi^{\Xi^-\to\Sigma^0K^{*-}}(u\to d) = \Pi_3(d,s,u) \ , \\
\Pi^{p\to\Sigma^0K^{*+}} - \sqrt{3}\Pi^{p\to\Lambda K^{*+}} &= 2\Pi^{\Xi^-\to\Sigma^0K^{*-}}(u\leftrightarrow s) = -2\Pi_3(s,u,d) \ , \\
\Pi^{n\to\Sigma^0\bar{K}^{*0}} + \sqrt{3}\Pi^{n\to\Lambda\bar{K}^{*0}} &= -2\Pi^{\Xi^-\to\Sigma^0K^{*-}}(u\leftrightarrow s)(d\leftrightarrow u) = 2\Pi_3(s,d,u) \ , \\
\Pi^{\Sigma^-\to\Sigma^0\rho^-} + \sqrt{3}\Pi^{\Sigma^-\to\Lambda\rho^-} &= 2\Pi^{\Xi^-\to\Sigma^0K^{*-}}(d\to s) = -2\Pi_3(u,d,s) \ , \\
\Pi^{\Sigma^+\to\Lambda\rho^+} &= \Pi^{\Sigma^-\to\Lambda\rho^-}(u\leftrightarrow d) \ , \\
\Pi^{\Sigma^0\to\Sigma^0\omega} &= \frac{1}{\sqrt{2}}\Big[\Pi_1(u,d,s) + \Pi_1(d,u,s)\Big] \ , \\
\Pi^{\Sigma^+\to\Sigma^+\omega} &= \sqrt{2}\Pi_1(u,u,s) \ , \\
\Pi^{\Sigma^-\to\Sigma^-\omega} &= \sqrt{2}\Pi_1(d,d,s) \ ,
\end{aligned}
$$

$$
\begin{aligned}
\Pi^{p\to p\omega} &= \sqrt{2}\Pi_1(u,u,d)+\frac{1}{\sqrt{2}}\Pi_2(u,u,d)\ , \\
\Pi^{n\to n\omega} &= \sqrt{2}\Pi_1(d,d,u)+\frac{1}{\sqrt{2}}\Pi_2(d,d,u)\ , \\
\Pi^{\Xi^0\to\Xi^0\omega} &= \frac{1}{\sqrt{2}}\Pi_2(s,s,u)\ , \\
\Pi^{\Xi^-\to\Xi^-\omega} &= \frac{1}{\sqrt{2}}\Pi_2(s,s,d)\ , \\
\Pi^{\Sigma^0\to\Lambda\omega}+\Pi^{\Lambda\to\Sigma^0\omega} &= \frac{2}{\sqrt{6}}\Big[\Pi_1(u,s,d)-\Pi_1(d,s,u)-\Pi_2(s,d,u)+\Pi_2(s,u,d)\Big]\ , \\
\Pi^{\Lambda\to\Lambda\omega} &= \frac{\sqrt{2}}{3}\Big[\Pi_1(u,s,d)+\Pi_1(d,s,u)+\Pi_2(s,d,u)+\Pi_2(s,u,d) \\
&- \frac{1}{2}\Pi_1(u,d,s)-\frac{1}{2}\Pi_1(d,u,s)\Big]\ , \\
\Pi^{\Sigma^0\to\Sigma^0\phi} &= \Pi_2(u,d,s)\ , \\
\Pi^{\Sigma^+\to\Sigma^+\phi} &= \Pi_2(u,u,s)\ , \\
\Pi^{\Sigma^-\to\Sigma^-\phi} &= \Pi_2(d,d,s)\ , \\
\Pi^{p\to p\phi} &= \Pi^{n\to n\phi}=0\ , \\
\Pi^{\Xi^0\to\Xi^0\phi} &= 2\Pi_1(s,s,u)\ , \\
\Pi^{\Xi^-\to\Xi^-\phi} &= 2\Pi_1(s,s,d)\ , \\
\Pi^{\Sigma^0\to\Lambda\phi}+\Pi^{\Lambda\to\Sigma^0\phi} &= \frac{2}{\sqrt{3}}\Big[-\Pi_1(s,d,u)+\Pi_1(s,u,d)\Big]\ , \\
\Pi^{\Lambda\to\Lambda\phi} &= \frac{2}{3}\Big[\Pi_1(s,d,u)+\Pi_1(s,u,d)-\frac{1}{2}\Pi_2(u,d,s)\Big]\ . \qquad (28)
\end{aligned}
$$

These relations allow us to express, all possible strong coupling constants of the octet vector mesons with the octet baryons in terms of three independent invariant functions without using the flavor symmetry.

Before starting to calculate these invariant functions from the QCD side we would like to make the following remark. The invariant function $\Pi_3(u,d,s)$ can be split into symmetric and antisymmetric parts with respect to the exchange of d and s quarks as:

$$
\Pi_3(u,d,s)=\Pi_3^{sym}(u,d,s)+\Pi_3^{asym}(u,d,s)\ .
$$

The symmetric part, Π_3^{sym}, can be expressed in terms of Π_1 and Π_2 as

$$
\Pi_3^{sym}(u,d,s)=\frac{1}{\sqrt{2}}\Big[\Pi_1(u,d,s)+\Pi_1(u,s,d)-\Pi_2(s,d,u)\Big]\ .
$$

The explicit form of $\Pi_3^{asym}(u,d,s)$, which vanishes in the $SU(3)_f$ limit, is given in the appendix along with the explicit forms of Π_1 and Π_2. Hence, in $SU(3)_f$ limit only two invariant functions Π_1 and Π_2 are relevant and they correspond to $\mathcal{F}$ and $\mathcal{D}-\mathcal{F}$ couplings. One more additional function Π_3^{asym} is needed in order to take $SU(3)_f$ violation into consideration.

We can now proceed to calculate these three invariant functions. As has already been mentioned, for this aim, the correlation functions responsible for the transitions $\Sigma^0 \to \Sigma^0 \rho^0$, $\Sigma^0 \to \Sigma^0 \phi$ and $\Sigma^0 \to \Xi^- K^{*+}$ are enough (see Eq. (4)).

In deep Euclidean region, $-p_1^2 \to \infty$, $-p_2^2 \to \infty$, the correlation function can be evaluated using OPE. In order to obtain the expressions of the correlation functions from QCD side, the propagator of the light quarks and the matrix elements of the nonlocal operators $\bar{q}(x_1)\Gamma q'(x_2)$ and $\bar{q}(x_1)G_{\mu\nu}q'(x_2)$ between the vacuum and the vector meson states are needed, where Γ represents the Dirac matrices relevant to the case under consideration, and $G_{\mu\nu}$ is the gluon field strength tensor.

Up to twist–4 accuracy, matrix elements $\langle V(q) \,|\bar{q}(x)\Gamma q(0)|\, 0\rangle$ and $\langle V(q) \,|\bar{q}(x)G_{\mu\nu} q(0)|\, 0\rangle$ are given in [12, 13] as follows:

$$
\begin{aligned}
\langle V(q,\lambda) \,|\bar{q}_1(x)\gamma_\mu q_2(0)|\, 0\rangle &= f_V m_V \Bigg\{ \frac{\varepsilon^\lambda\!\cdot\! x}{q\!\cdot\! x} q_\mu \int_0^1 du e^{i\bar{u}qx} \left[\phi_\parallel(u) + \frac{m_V^2 x^2}{16} A_\parallel(u) \right] \\
&+ \left(\varepsilon_\mu^\lambda - q_\mu \frac{\varepsilon^\lambda\!\cdot\! x}{q\!\cdot\! x} \right) \int_0^1 du e^{i\bar{u}qx} g_\perp^v(u) \\
&- \frac{1}{2} x_\mu \frac{\varepsilon^\lambda\!\cdot\! x}{(q\!\cdot\! x)^2} m_V^2 \int_0^1 du e^{i\bar{u}qx} \left[g_3(u) + \phi_\parallel(u) - 2 g_\perp^v(u) \right] \Bigg\} ~, \\
\langle V(q,\lambda) \,|\bar{q}_1(x)\gamma_\mu \gamma_5 q_2(0)|\, 0\rangle &= -\frac{1}{4} \epsilon_\mu^{\nu\alpha\beta} \varepsilon^\lambda q_\alpha x_\beta f_V m_V \int_0^1 du e^{i\bar{u}qx} g_\perp^a(u) ~, \\
\langle V(q,\lambda) \,|\bar{q}_1(x)\sigma_{\mu\nu} q_2(0)|\, 0\rangle &= -i f_V^T \Bigg\{ (\varepsilon_\mu^\lambda q_\nu - \varepsilon_\nu^\lambda q_\mu) \int_0^1 du e^{i\bar{u}qx} \left[\phi_\perp(u) + \frac{m_V^2 x^2}{16} A_\perp(u) \right] \\
&+ \frac{\varepsilon^\lambda\!\cdot\! x}{(q\!\cdot\! x)^2} (q_\mu x_\nu - q_\nu x_\mu) \int_0^1 du e^{i\bar{u}qx} \left[h_\parallel^t - \frac{1}{2}\phi_\perp - \frac{1}{2} h_3(u) \right] \\
&+ \frac{1}{2} (\varepsilon_\mu^\lambda x_\nu - \varepsilon_\nu^\lambda x_\mu) \frac{m_V^2}{q\!\cdot\! x} \int_0^1 du e^{i\bar{u}qx} \Big[h_3(u) - \phi_\perp(u) \Big] \Bigg\} ~, \\
\langle V(q,\lambda) \,|\bar{q}_1(x)\sigma_{\alpha\beta} g G_{\mu\nu}(ux) q_2(0)|\, 0\rangle &= f_V^T m_V^2 \frac{\varepsilon^\lambda\!\cdot\! x}{2 q\!\cdot\! x} \Big[q_\alpha q_\mu g_{\beta\nu}^\perp - q_\beta q_\mu g_{\alpha\nu}^\perp - q_\alpha q_\nu g_{\beta\mu}^\perp + q_\beta q_\nu g_{\alpha\mu}^\perp \Big] \\
&\times \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + u\alpha_g)qx} \mathcal{T}(\alpha_i) \\
&+ f_V^T m_V^2 \Big[q_\alpha \varepsilon_\mu^\lambda g_{\beta\nu}^\perp - q_\beta \varepsilon_\mu^\lambda g_{\alpha\nu}^\perp - q_\alpha \varepsilon_\nu^\lambda g_{\beta\mu}^\perp + q_\beta \varepsilon_\nu^\lambda g_{\alpha\mu}^\perp \Big] \\
&\times \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + u\alpha_g)qx} \mathcal{T}_1^{(4)}(\alpha_i) \\
&+ f_V^T m_V^2 \Big[q_\mu \varepsilon_\alpha^\lambda g_{\beta\nu}^\perp - q_\mu \varepsilon_\beta^\lambda g_{\alpha\nu}^\perp - q_\nu \varepsilon_\alpha^\lambda g_{\beta\mu}^\perp + q_\nu \varepsilon_\beta^\lambda g_{\alpha\mu}^\perp \Big] \\
&\times \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + u\alpha_g)qx} \mathcal{T}_2^{(4)}(\alpha_i) \\
&+ \frac{f_V^T m_V^2}{q\!\cdot\! x} \Big[q_\alpha q_\mu \varepsilon_\beta^\lambda x_\nu - q_\beta q_\mu \varepsilon_\alpha^\lambda x_\nu - q_\alpha q_\nu \varepsilon_\beta^\lambda x_\mu + q_\beta q_\nu \varepsilon_\alpha^\lambda x_\mu \\
&\times \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + u\alpha_g)qx} \mathcal{T}_3^{(4)}(\alpha_i)
\end{aligned}
$$

$$
\begin{aligned}
&+ \frac{f_V^T m_V^2}{q\cdot x}\Big[q_\alpha q_\mu \varepsilon_\nu^\lambda x_\beta - q_\beta q_\mu \varepsilon_\nu^\lambda x_\alpha - q_\alpha q_\nu \varepsilon_\mu^\lambda x_\beta + q_\beta q_\nu \varepsilon_\mu^\lambda x_\alpha \\
&\times \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar q}+u\alpha_g)qx}\mathcal{T}_4^{(4)}(\alpha_i)\ ,
\end{aligned}
$$

$$
\begin{aligned}
\langle V(q,\lambda)\,|\bar q_1(x) g_s G_{\mu\nu}(ux) q_2(0)|\,0\rangle &= -i f_V^T m_V (\varepsilon_\mu^\lambda q_\nu - \varepsilon_\nu^\lambda q_\mu)\int \mathcal{D}\alpha_i e^{i(\alpha_{\bar q}+u\alpha_g)qx}\mathcal{S}(\alpha_i)\ , \\
\left\langle V(q,\lambda)\left|\bar q_1(x) g_s \widetilde G_{\mu\nu}(ux)\gamma_5 q_2(0)\right| 0\right\rangle &= -i f_V^T m_V (\varepsilon_\mu^\lambda q_\nu - \varepsilon_\nu^\lambda q_\mu)\int \mathcal{D}\alpha_i e^{i(\alpha_{\bar q}+u\alpha_g)qx}\widetilde{\mathcal{S}}(\alpha_i)\ , \\
\left\langle V(q,\lambda)\left|\bar q_1(x) g_s \widetilde G_{\mu\nu}(ux)\gamma_\alpha\gamma_5 q_2(0)\right| 0\right\rangle &= f_V m_V q_\alpha (\varepsilon_\mu^\lambda q_\nu - \varepsilon_\nu^\lambda q_\mu)\int \mathcal{D}\alpha_i e^{i(\alpha_{\bar q}+u\alpha_g)qx}\mathcal{A}(\alpha_i)\ , \\
\langle V(q,\lambda)\,|\bar q_1(x) g_s G_{\mu\nu}(ux) i\gamma_\alpha q_2(0)|\,0\rangle &= f_V m_V q_\alpha (\varepsilon_\mu^\lambda q_\nu - \varepsilon_\nu^\lambda q_\mu)\int \mathcal{D}\alpha_i e^{i(\alpha_{\bar q}+u\alpha_g)qx}\mathcal{V}(\alpha_i)\ , \qquad (29)
\end{aligned}
$$

where $\widetilde G_{\mu\nu} = (1/2)\epsilon_{\mu\nu\alpha\beta}G^{\alpha\beta}$ is the dual gluon field strength tensor, and $\int \mathcal{D}\alpha_i = \int d\alpha_q d\alpha_{\bar q} d\alpha_g \delta(1-\alpha_q-\alpha_{\bar q}-\alpha_g)$. In further analysis, we use the following expression for the light quark propagator

$$
\begin{aligned}
S_q(x) &= \frac{i\not x}{2\pi^2 x^4} - \frac{m_q}{4\pi^2 x^2} - \frac{\langle \bar q q\rangle}{12}\left(1-\frac{im_q}{4}\not x\right) - \frac{x^2}{192}m_0^2\langle \bar q q\rangle\left(1-\frac{im_q}{6}\not x\right) \\
&- i g_s\int_0^1 du\Bigg\{\frac{\not x}{16\pi^2 x^2}G_{\mu\nu}(ux)\sigma^{\mu\nu} - ux^\mu G_{\mu\nu}(ux)\gamma^\nu\frac{i}{4\pi^2 x^2} \\
&- \frac{im_q}{32\pi^2}G_{\mu\nu}(ux)\sigma^{\mu\nu}\left[\ln\left(\frac{-x^2\Lambda^2}{4}\right)+2\gamma_E\right]\Bigg\}\ , \qquad (30)
\end{aligned}
$$

where γ_E is the Euler constant, Λ is a scale parameter, and we will choose it as a factorization scale, i.e., $\Lambda = 0.5 \div 1.0\ GeV$ (for more detail, see [14]). Note that, in the calculations, $SU(3)_f$ symmetry violation effects are included in the nonzero strange quark mass and different strange quark condensate. These effects are also taken into account in calculation of the distribution amplitudes [12, 13].

The expressions of the full propagator of the light quark and the definition of the distribution amplitudes allow us to calculate the theoretical part of the correlation functions. Equating both representations of correlation function and separating coefficients of Lorentz structures $\not p\not\varepsilon\not q$ and $\not p(\varepsilon\cdot p)$ and applying Borel transformation to both side of the correlation functions on the variables p^2 and $(p+q)^2$ in order to suppress the contributions of the higher states and continuum (see [17]), we get the sum rules for the corresponding vector meson baryon couplings. The contributions of higher states and the continuum are subtracted using quark-hadron duality. After lengthy calculations, for each Lorentz structure the expressions for the three invariant functions $\Pi_i^{(\alpha)}$, $i=1,2,3$ are obtained and their expressions are presented in Appendix–A. Here superscript α refers to the invariant functions $\Pi_i^{(\alpha)}$ relevant to the coupling constants f_1 and f_1+f_2, respectively.

For a given transition $B_1 \to B_2 V$, once the Borel transformed and continuum subtracted coefficient functions Π^{f_1} and $\Pi^{f_1+f_2}$ are obtained, the sum rules for the electric and magnetic

type couplings are obtained as

$$f_1 = \frac{1}{\lambda_{B_1}\lambda_{B_2}} e^{-\frac{m_1^2}{M_1^2}-\frac{m_2^2}{M_2^2}-\frac{m_V^2}{M_1^2+M_2^2}} \Pi^{f_1}$$

$$f_1 + f_2 = \frac{1}{\lambda_{B_1}\lambda_{B_2}} e^{-\frac{m_1^2}{M_1^2}-\frac{m_2^2}{M_2^2}-\frac{m_V^2}{M_1^2+M_2^2}} \Pi^{f_1+f_2}$$

In determining the vector meson–octet baryon strong coupling constants, the residues of baryons are needed. The residues of baryons are obtained from the analysis of two–point correlation function are given in [15–17]. The currents of the other baryons can be obtained from Σ^0 current by making appropriate substitutions of quarks. For this reason, for determination of the residues, we give the sum rule only for Σ^0

$$\begin{aligned}
\lambda_{\Sigma^0}^2 e^{-m_{\Sigma^0}^2/M^2} = \; & \frac{M^6}{1024\pi^2}(5+2\beta+5\beta^2) - \frac{m_0^2}{96M^2}(-1+\beta)^2 \langle \bar{u}u\rangle \langle \bar{d}d\rangle \\
& - \frac{m_0^2}{16M^2}(-1+\beta^2)\langle \bar{s}s\rangle \Big(\langle \bar{u}u\rangle + \langle \bar{d}d\rangle \Big) \\
& + \frac{3m_0^2}{128}(-1+\beta^2)\Big[m_s\Big(\langle \bar{u}u\rangle + \langle \bar{d}d\rangle \Big) + (m_u+m_d)\langle \bar{s}s\rangle \Big] \\
& - \frac{1}{64\pi^2}(-1+\beta)^2 M^2 \Big(m_d \langle \bar{u}u\rangle + m_s \langle \bar{d}d\rangle \Big) \\
& - \frac{3M^2}{64\pi^2}(-1+\beta^2)\Big[m_s\Big(\langle \bar{u}u\rangle + \langle \bar{d}d\rangle \Big) + (m_u+m_d)\langle \bar{s}s\rangle \Big] \\
& + \frac{1}{128\pi^2}(5+2\beta+5\beta^2)\Big(m_u \langle \bar{u}u\rangle + m_d \langle \bar{d}d\rangle + m_s \langle \bar{s}s\rangle \Big) \\
& + \frac{1}{24}\Big[3(-1+\beta^2)\langle \bar{s}s\rangle \Big(\langle \bar{u}u\rangle + \langle \bar{d}d\rangle \Big) + (-1+\beta^2)\langle \bar{u}u\rangle \langle \bar{d}d\rangle \Big] \\
& + \frac{m_0^2}{256\pi^2}(-1+\beta)^2\Big(m_u \langle \bar{d}d\rangle + m_d \langle \bar{u}u\rangle \Big) \\
& + \frac{m_0^2}{26\pi^2}(-1+\beta^2)\Big[13 m_s\Big(\langle \bar{u}u\rangle + \langle \bar{d}d\rangle \Big) + 11(m_u+m_d)\langle \bar{s}s\rangle \Big] \\
& - \frac{m_0^2}{192\pi^2}(1+\beta+\beta^2)\Big(m_u \langle \bar{u}u\rangle + m_d \langle \bar{d}d\rangle - 2m_s \langle \bar{s}s\rangle \Big) \; . \qquad (31)
\end{aligned}$$

It follows from Eq. (31) that, sum rules cannot predict the sign of the residue. We have chosen the sign convention such that in the $SU(3)_f$ symmetry, the signs correctly reproduce the $\mathcal{F}$ and $\mathcal{D}$ couplings (see [7]).

4 Numerical analysis and discussion

This section is devoted to the numerical analysis of the sum rules for the vector meson–octet baryon coupling constants. The main input parameters of the light cone sum rules in our case are the vector meson distribution amplitudes (DAs). The DAs of the vector mesons are given in [12–14]. The values of the leptonic constants f_V and f_V^T, and of the twist–2 and twist–3 parameters $a_i^\parallel$, $a_i^\perp$,$\zeta_{3V}^\parallel$, $\tilde{\lambda}_{3V}^\parallel$, $\tilde{\omega}_{3V}^\parallel$, $\kappa_{3V}^\parallel$, $\omega_{3V}^\parallel$, $\lambda_{3V}^\parallel$, $\kappa_{3V}^\perp$, $\omega_{3V}^\perp$, $\lambda_{3V}^\perp$, as

well as twist–4 parameters $\zeta_4^{\parallel}$, $\tilde{\omega}_4^{\parallel}$, $\zeta_4^{\perp}$, $\tilde{\zeta}_4^{\perp}$, $\kappa_{4V}^{\parallel}$, $\kappa_{4V}^{\perp}$ are given in Table (1) and Table (2), respectively, in [12]. The value of the other input parameters which are needed in the sum rule are $\langle \bar{q}q \rangle = -(0.243\ GeV)^3$, $m_0^2 = 0.8$ [15], $\langle g_s^2 G^2 \rangle = 0.47\ GeV^4$ [3].

In the problem under consideration, the masses of initial and final baryons are, more or less, equal to each other. For this reason we choose $M_1^2 = M_2^2 = 2M^2$, and consequently we set $u_0 = 1/2$. Hence, in further numerical analysis, the values of the DAs only at $u_0 = 1/2$ are needed.

It follows from the explicit expressions of the sum rules for the vector meson–octet baryon coupling constants that, in addition to the DAs, they also contain three auxiliary parameters, namely, Borel mass parameter, continuum threshold s_0, and the parameter β in the interpolating current. Since any physically measurable quantity should be independent of them, we need to look for regions of M^2, s_0, and in which the results of the vector meson–octet baryon coupling constants are practically independent of these parameters.

The upper bound for the Borel parameter M^2 is determined by demanding that the higher states and continuum contributions to a correlation function should be less than half the value of the same correlation function. The lower bound of M^2 can be determined by requiring that the contribution of the highest term with the power of $1/M^2$ be less than 25%. Using these restrictions, we obtain the working region for the Borel parameters. The continuum threshold is varied in the regions $s_0 = (m_B + 0.5)^2$ and $s_0 = (m_B + 0.7)^2$.

To demonstrate the analysis, in Figs. 1 and 2, we depict the dependence of $f_1^{p\to p\rho_0}$ and $f_1^{p\to p\rho_0} + f_2^{p\to p\rho_0}$ on M^2 at three different values of the parameter β, and at two fixed values of s_0. The results for $f_1^{p\to p\rho_0}$ and $f_1^{p\to p\rho_0} + f_2^{p\to p\rho_0}$ exhibited by these figures show good stability with respect to the variation of M^2 in its working domain. As has already been noted, the sum rules contain another arbitrary parameter β, and with similar reasoning, we should find such region of β, in which the results for the coupling constants are independent of it. For this purpose, in Figs. 3 and 4, we present the dependence $f_1^{p\to p\rho_0}$ and $f_1^{p\to p\rho_0} + f_2^{p\to p\rho_0}$ on $\cos\theta$, where θ is defined as $\tan\theta = \beta$. From these figures one can conclude that, the working region for the unphysical parameter β is $-0.5 < \cos\theta < 0.3$ for $f_1^{p\to p\rho_0}$, and $-0.7 < \cos\theta < 0.1$ for $f_1^{p\to p\rho_0} + f_2^{p\to p\rho_0}$, where the coupling constants $f_1^{p\to p\rho_0}$ and $f_1^{p\to p\rho_0} + f_2^{p\to p\rho_0}$ are insensitive to the variation of β. As a result of these considerations, we find that $f_1^{p\to p\rho_0} = -2.9 \pm 0.9$ and $f_2^{p\to p\rho_0} = 19.7 \pm 2.8$.

Performing similar analysis, the results for the other coupling constants of vector mesons with octet baryons are presented in Table 1. For completeness, we also present the existing results in literature in the same Table. Note that in this Table we present only those results which are not obtained from each other by a simple $SU(2)$ and isotopic spin relations. We also would like to remind that, the signs of the residues are not fixed by the sum rules. this leaves an ambiguity in the signs of any seven (since an overall sign does not effect the coupling) couplings. These signs have already been fixed in [7] to follow the $SU(3)_f$ symmetry. In this work we follow the same sign convention. The error bars in the table take into account only the uncertainties due to the variations of the auxiliary parameters and the uncertainties in the input parameters.

From the results summarized in Table (1), we can comment as follows:

a) For the coupling f_1: A comparison of the predictions on the coupling constants which are obtained using the most general form of the currents, with the ones obtained using the Ioffe current shows substantial difference for many channels. For example, the coupling

$f_1$channel	General current Result	General current $SU(3)_f$	Ioffe current Result	Ioffe current $SU(3)_f$	QSR [4]	QSR [5]	QSR [6]
$f_1^{p\to p\rho^0}$	-2.5 ± 1.1	-1.7	-5.9 ± 1.3	-6.4	2.5 ± 0.2	2.4 ± 0.6	3.2 ± 0.9
$f_1^{p\to p\omega}$	-8.9 ± 1.5	-10.3	-8.2 ± 0.4	-9.6	18 ± 8	7.2 ± 1.8	—
$f_1^{\Xi^0\to\Xi^0\rho^0}$	-4.2 ± 2.1	-4.3	-2.0 ± 0.2	-1.6	—	2.4 ± 0.6	1.5 ± 1.1
$f_1^{\Sigma^0\to\Lambda\rho^0}$	1.9 ± 0.7	1.5	-3.0 ± 0.5	-2.8	—	—	—
$f_1^{\Lambda\to\Sigma^+\rho^-}$	1.9 ± 0.7	1.5	-2.8 ± 0.6	-2.8	—	—	—
$f_1^{\Sigma^+\to\Sigma^0\rho^+}$	7.2 ± 1.2	6.0	8.5 ± 0.8	8.0	—	—	—
$f_1^{\Sigma^+\to\Lambda\rho^+}$	2.0 ± 0.6	1.5	-2.8 ± 0.6	-2.8	—	—	—
$f_1^{p\to\Lambda K^{*+}}$	5.1 ± 1.8	4.4	7.4 ± 0.8	8.3	—	—	—
$f_1^{\Sigma^-\to nK^{*-}}$	6.6 ± 1.8	6.1	1.7 ± 0.4	2.3	—	—	—
$f_1^{\Xi^0\to\Sigma^+K^{*-}}$	-2.3 ± 1.7	-2.4	-10.0 ± 1.8	-9.1	—	—	—
$f_1^{\Xi^-\to\Lambda K^{*-}}$	-5.9 ± 0.7	-5.8	-6.2 ± 0.4	-5.5	—	—	—
$f_1^{\Sigma^0\to\Xi^0K^{*0}}$	1.6 ± 1.0	1.7	7.1 ± 1.3	6.4	—	—	—
$f_1^{\Lambda\to\Xi^0K^{*0}}$	-6.0 ± 0.7	-5.9	-6.2 ± 0.2	-5.5	—	—	—
$f_1^{n\to\Sigma^0K^{*0}}$	-4.0 ± 0.7	-4.3	-1.5 ± 0.3	-1.6	—	—	—
$f_1^{\Lambda\to\Lambda\omega}$	-7.1 ± 1.1	-7.7	-4.8 ± 0.2	-4.8	—	4.8 ± 1.2	—
$f_1^{\Xi^0\to\Xi^0\phi}$	-9.5 ± 2.5	-8.5	-13.5 ± 1.6	-11.3	—	—	—
$f_1^{\Lambda\to\Lambda\phi}$	-5.3 ± 1.5	-3.6	-8.0 ± 1.0	-6.8	—	—	—
$f_1^{\Sigma^0\to\Sigma^0\phi}$	-6.0 ± 0.8	-6.1	-0.25 ± 0.50	-2.3	—	—	—

Table 1: The values of the electric coupling constants for various channels.

constants f_1 for the $p\to\Sigma^+K^{*0}$, $\Xi^0\to\Sigma^0\bar{K}^{*0}$, $\Sigma^-\to nK^{*-}$, $n\to p\rho^-$, $\Xi^0\to\Sigma^-K^{*-}$, $\Sigma^0\to\Xi^0K^{*0}$ channels for the general case, differ considerably from the prediction of the Ioffe current. Especially, the difference between the predictions of the above–mentioned currents for the $\Sigma^0\to\Sigma^0\phi$ transition is worth mentioning. While the Ioffe current predicts $f_1\simeq 0$, the general current case predicts $f_1\simeq -5$. The sign of f_1 for the $\Sigma^0\to\Lambda\rho^0$, $\Sigma^+\to\Lambda\rho^+$, transitions differ from those predicted by the Ioffe current. Our predictions on the coupling constant f_1 for the $p\to p\rho^0$ within their error limits, are closer to the results predicted by [4], [5] and [6]. There is considerable difference for the $p\to p\omega$ transition between our result compared to that obtained in [4], but our result is close to the results of [5].

b) For the coupling f_1+f_2: Except the $\Xi^0\to\Xi^0\rho^0$, $\Sigma^+\to\Sigma^0\rho^+$, $\Sigma^-\to nK^{*-}$, $\Xi^0\to\Xi^0\phi$, $n\to\Sigma^0K^{*0}$, $p\to\Sigma^+K^{*0}$, $\Lambda\to\Lambda\omega$ and $\Sigma^0\to\Sigma^0\phi$ transitions, our predictions for the general current is in good agreement with the predictions of Ioffe current.

These discrepancies between the coupling constants obtained using the general form of the baryon current and the Ioffe current can be explained as follows. For many channels the value $\beta=-1$ lies outside the stability region of β, as a result of which considerable

$(f_1+f_2)^{\text{channel}}$	General current Result	General current $SU(3)_f$	Ioffe current Result	Ioffe current $SU(3)_f$	QSR [4]	QSR [5]	QSR [6]
$(f_1+f_2)^{p\to p\rho^0}$	19.7±2.8	21.4	22.7±1.3	24.7	21.6 ± 6.6	10.1 ± 3.7	36.8 ± 13
$(f_1+f_2)^{p\to p\omega}$	14.5±2.6	15.0	21.2±1.2	25.7	32.4 ± 14.4	5.0 ± 1.2	—
$(f_1+f_2)^{\Xi^0\to\Xi^0\rho^0}$	−2.8±1.6	−3.2	−0.24±0.24	0.5	—	-3.6 ± 1.6	-5.3 ± 3.3
$(f_1+f_2)^{\Sigma^0\to\Lambda\rho^0}$	13.8±2.7	14.2	15.1±0.9	14.0	—	—	—
$(f_1+f_2)^{\Lambda\to\Sigma^+\rho^-}$	14.3±2.9	14.2	15.1±0.8	14.0	—	—	—
$(f_1+f_2)^{\Sigma^+\to\Sigma^0\rho^+}$	−17.8±2.2	−18.2	−27.9±1.8	−25.2	—	7.1 ± 1.0	53.5 ± 19
$(f_1+f_2)^{\Sigma^+\to\Lambda\rho^+}$	14.3±2.9	14.2	15.1±0.8	14.0	—	—	—
$(f_1+f_2)^{p\to\Lambda K^{*+}}$	−22.9±4.2	−22.9	−27.3±1.5	−28.8	—	—	—
$(f_1+f_2)^{\Sigma^-\to nK^{*-}}$	3.8±2.8	4.5	−0.79±0.05	−0.7	—	—	—
$(f_1+f_2)^{\Xi^0\to\Sigma^+K^{*-}}$	33.8±4.9	30.3	41.3±2.4	34.9	—	—	—
$(f_1+f_2)^{\Xi^-\to\Lambda K^{*-}}$	11.6±2.9	8.7	17.9±1.0	14.8	—	—	—
$(f_1+f_2)^{\Sigma^0\to\Xi^0K^{*0}}$	−24.6±4.8	−21.4	−29.2±1.7	−24.7	—	—	—
$(f_1+f_2)^{\Lambda\to\Xi^0K^{*0}}$	11.1±2.6	8.7	15.0±1.0	14.8	—	—	—
$(f_1+f_2)^{n\to\Sigma^0K^{*0}}$	−2.8±1.8	−3.2	0.56±0.04	0.5	—	—	—
$(f_1+f_2)^{\Lambda\to\Lambda\omega}$	1.6±0.6	1.8	7.1±0.5	9.1	—	-5.7 ± 1.0	—
$(f_1+f_2)^{\Xi^0\to\Xi^0\phi}$	22.8±6.4	25.7	37.7±2.5	35.6	—	—	—
$(f_1+f_2)^{\Lambda\to\Lambda\phi}$	19.3±5.0	18.7	22.0±1.4	23.5	—	—	—
$(f_1+f_2)^{\Sigma^0\to\Sigma^0\phi}$	−3.5±2.5	4.5	0.81±0.05	0.7	—	—	—

Table 2: The values of the magnetic coupling constants for various channels.

differences appear between the predictions of the above–mentioned baryon currents, making the predictions less reliable.

In the Tables, we have also presented in the columns labeled $SU(3)_f$ the best fits to our results of the $SU(3)_f$ expressions given in Eq. (1). The $SU(3)_f$ fits in Table–1 corresponds to the central values of $\mathcal{F}=-3.0\pm0.5$, $\mathcal{D}=1.3\pm0.6$ and $\mathcal{F}=-4.2\pm0.7$, $\mathcal{D}=-2.7\pm1.0$ for the general and Ioffe current, respectively. For the central values, these yield $\alpha_E=1.6$ and $\alpha_E=0.61$, respectively, both of which deviates from VDM model prediction $\alpha_E=1$ considerably.

In Table–2, the $SU(3)_f$ fit value corresponds to $\mathcal{F}=9.2\pm1.0$, $\mathcal{D}=12.4\pm1.4$ and $\mathcal{F}=12.7\pm1.8$, $\mathcal{D}=12.2\pm1.8$ for the general form of the baryon current and $\beta=-1$ baryon current, respectively. For the α_M value of the magnetic type coupling, these predictions yield $\alpha_M=0.43$ and $\alpha_M=0.85$, respectively. α_M is also calculated in [18] using the soft core potential and it is predicted to be $\alpha_M=0.44$, in agreement with the prediction of the general current.

In conclusion, the strong coupling constants of the vector mesons with octet baryons are investigated in LCSR. It is proven that all coupling constants can be written in terms

of three universal functions, which at exact $SU(3)_f$ symmetry case reduces to $\mathcal{F}$ and $\mathcal{D}$ couplings. The numerical values of the electric and magnetic couplings are obtained.

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Appendix A :

In this appendix we present the explicit expressions of the six Borel transformed invariant functions.

Electric Type Coupling

The electric type coupling is determined by the coefficient of the structure $\not p \not e \not q$:

$$
\begin{aligned}
& e^{m_V^2/4M^2}\Pi_1^{f_1}(u,d,s) = -\frac{1}{96\pi^2}M^4(1+\beta^2)f_V^{\parallel}m_V\phi_V^{\parallel}(u_0) \\
&+ \frac{1}{384M^2\pi^2}(1-\beta)f_V^{\perp}m_V^2\left(\gamma_E - \ln\frac{M^2}{\Lambda^2}\right)\Big\{\langle g_s^2G^2\rangle[-m_d(1-\beta) \\
&+ 3m_s(1+\beta)]\psi_{3;V}^{\parallel}(u_0) + 4M^2m_V^2\Big([-m_d(1-\beta)-9m_s(1+\beta)]i_1(\mathcal{T},1) \\
&+ 4m_s(1+\beta)i_1(\mathcal{T}_3+2\mathcal{T}_4,1)\Big)\Big\} \\
&+ \frac{1}{192M^8}(1-\beta^2)f_V^{\parallel}m_V^3\Big[\langle g_s^2G^2\rangle m_0^2(m_s\langle\bar dd\rangle + m_d\langle\bar ss\rangle)\widetilde{\widetilde{i}}_4(\mathbb{C})\Big] \\
&+ \frac{1}{288M^6}(1-\beta^2)f_V^{\parallel}m_V^3\Big[3\langle g_s^2G^2\rangle(m_s\langle\bar dd\rangle + m_d\langle\bar ss\rangle)\widetilde{\widetilde{i}}_4(\mathbb{C}) \\
&+ 4m_0^2m_V^2(m_s\langle\bar dd\rangle - m_d\langle\bar ss\rangle)i_0(\widetilde{\Psi},1)\Big] \\
&+ \frac{1}{1152M^4\pi^2}m_V^4f_V^{\perp}\langle g_s^2G^2\rangle(1-\beta)[m_d(1-\beta)+m_s(1+\beta)]i_1(\mathcal{T}-\mathcal{T}_4,1) \\
&+ \frac{1}{72M^4}m_V^3f_V^{\parallel}(1-\beta^2)\Big[3m_0^2(m_s\langle\bar dd\rangle + m_d\langle\bar ss\rangle)\widetilde{\widetilde{i}}_4(\mathbb{C}) + 4m_V^2(m_s\langle\bar dd\rangle - m_d\langle\bar ss\rangle)i_0(\widetilde{\Psi},1)\Big] \\
&+ \frac{1}{288M^2}m_V^3f_V^{\parallel}\Big\{2[m_s\langle\bar ss\rangle(1+\beta)^2 - m_d\langle\bar dd\rangle(3\beta^2+2\beta+3)]\Big[i_2(\mathcal{A},1) - i_2(\mathcal{V},1-2v)\Big] \\
&- 8[m_s\langle\bar ss\rangle\beta - m_d\langle\bar dd\rangle(2\beta^2+\beta+2)]i_2(\Phi,1-2v) + 4[m_s\langle\bar ss\rangle(1+\beta^2) \\
&- 2m_d\langle\bar dd\rangle(2\beta^2+\beta+2)]i_2(\widetilde{\Phi},1) + 4(m_d\langle\bar dd\rangle + m_s\langle\bar ss\rangle)(1+\beta^2)i_2(\Psi,1-2v) \\
&- 4[2m_s\langle\bar ss\rangle\beta + m_d\langle\bar dd\rangle(1+\beta^2)]i_2(\widetilde{\Psi},1) + 12[6(1-\beta^2)(m_d\langle\bar ss\rangle + m_s\langle\bar dd\rangle) \\
&+ (m_d\langle\bar dd\rangle + m_s\langle\bar dd\rangle)(5\beta^2+2\beta+5)]\widetilde{\widetilde{i}}_4(\mathbb{C}) + 3(m_d\langle\bar dd\rangle + m_s\langle\bar ss\rangle)(1+\beta^2)\mathbb{A}(u_0)\Big\} \\
&+ \frac{1}{432M^2}m_0^2m_Vf_V^{\parallel}(m_d\langle\bar dd\rangle + m_s\langle\bar ss\rangle)(3\beta^2+2\beta+3)\phi_V^{\parallel}(u_0) \\
&+ \frac{1}{36M^2}m_V^4f_V^{\perp}(1-\beta)[\langle\bar dd\rangle(1-\beta) + \langle\bar ss\rangle(1+\beta)]i_1(\mathcal{T}-\mathcal{T}_4,1) \\
&+ \frac{1}{576M^2\pi^2}m_V^2f_V^{\perp}\Big\{\langle g_s^2G^2\rangle[m_d(1-\beta)^2 - 3m_s(1-\beta^2)] \\
&- 8m_0^2\pi^2[\langle\bar dd\rangle(1-\beta)^2 - 4\langle\bar ss\rangle(1-\beta^2)]\Big\}\psi_{3;V}^{\parallel}(u_0) \\
&+ \frac{1}{384\pi^2}m_V^2M^2\Big\{(1+\beta^2)f_V^{\parallel}\Big(3m_V\mathbb{A}(u_0) - 4m_Vi_2(\mathcal{A},1) + 4m_V[i_2(4\Phi+2\Psi+\mathcal{V},1-2v)\Big) \\
&- 4m_Vf_V^{\parallel}\Big[(3\beta^2+2\beta+3)i_2(\widetilde{\Phi},1) + (1+\beta)^2i_2(\widetilde{\Psi},1) - 3(5\beta^2+2\beta+5)\widetilde{\widetilde{i}}_4(\mathbb{C})\Big]
\end{aligned}
$$

$$
\begin{aligned}
&- 12 f_V^{\perp}(1-\beta)\Big[m_d(1-\beta) - 3m_s(1+\beta)\Big]\psi_{3;V}^{\parallel}(u_0)\Big\}\\
&- \frac{1}{24} m_V \Big\{ f_V^{\parallel}(m_d\langle\bar{d}d\rangle + m_s\langle\bar{s}s\rangle)(1+\beta^2)\phi_V^{\parallel}(u_0)\\
&- 2m_V f_V^{\perp}(1-\beta)[\langle\bar{d}d\rangle(1-\beta) - 3\langle\bar{s}s\rangle(1+\beta)]\psi_{3;V}^{\parallel}(u_0)\Big\}\\
&- \frac{1}{12\pi^2}(1+\beta^2) m_V^5 f_V^{\parallel}\Big[i_0(\Psi, 1-2v) - i_0(\widetilde{\Psi}, 1)\Big]\\
&- \frac{1}{96\pi^2} m_V^4 f_V^{\perp}(1-\beta)\Big\{[3m_d(1-\beta) + 11m_s(1+\beta)]i_1(\mathcal{T}, 1) - 4m_s(1+\beta)i_1(\mathcal{T}_3, 1)\\
&- 2[m_d(1-\beta) + 5m_s(1+\beta)]i_1(\mathcal{T}_4, 1)\Big\} \;, \qquad\qquad \text{(A.1)}
\end{aligned}
$$

$$
\begin{aligned}
&e^{m_V^2/4M^2}\Pi_2^{f_1}(u,d,s) = -\frac{1}{96\pi^2} M^4(3\beta^2 + 2\beta + 3) f_V^{\parallel} m_V \phi_V^{\parallel}(u_0)\\
&- \frac{1}{384 M^2\pi^2}(m_d + m_u)(1-\beta^2) f_V^{\perp} m_V^2\left(\gamma_E - \ln\frac{M^2}{\Lambda^2}\right)\Big[4M^2 m_V^2 i_1(9\mathcal{T} - 8\mathcal{T}_3 - 4\mathcal{T}_4, 1)\\
&- 3\langle g_s^2 G^2\rangle\psi_{3;V}^{\parallel}(u_0)\Big]\\
&- \frac{1}{576 M^8}\langle g_s^2 G^2\rangle f_V^{\parallel} m_0^2 m_V^3(1-\beta)^2(m_u\langle\bar{d}d\rangle + m_d\langle\bar{u}u\rangle)\widetilde{\widetilde{i}}_4(\mathbb{C})\\
&- \frac{1}{288 M^6} f_V^{\parallel} m_V^3(1-\beta)^2(m_u\langle\bar{d}d\rangle + m_d\langle\bar{u}u\rangle)\Big[\langle g_s^2 G^2\rangle\widetilde{\widetilde{i}}_4(\mathbb{C}) + 4m_0^2 m_V^2 i_0(\widetilde{\Psi}, 1)\Big]\\
&- \frac{1}{1152 M^4\pi^2} m_V^4 f_V^{\perp}\langle g_s^2 G^2\rangle(1-\beta^2)(m_u + m_d) i_1(\mathcal{T} - \mathcal{T}_4, 1)\\
&- \frac{1}{18 M^4} m_V^5 f_V^{\parallel}(1-\beta)^2(m_u\langle\bar{d}d\rangle + m_d\langle\bar{u}u\rangle) i_0(\widetilde{\Psi}, 1)\\
&- \frac{1}{288 M^2} m_V^3 f_V^{\parallel}(m_u\langle\bar{u}u\rangle + m_d\langle\bar{d}d\rangle)(3\beta^2 + 2\beta + 3)\Big[2i_2(\mathcal{A}, 1) - 2i_2(4\Phi + 2\Psi + \mathcal{V}, 1-2v)\\
&- 3\mathbb{A}(u_0)\Big]\\
&- \frac{1}{72 M^2} m_V^3 f_V^{\parallel}\Big\{3[2(1-\beta)^2(m_u\langle\bar{d}d\rangle + m_d\langle\bar{u}u\rangle) - (m_u\langle\bar{u}u\rangle + m_d\langle\bar{d}d\rangle)(5\beta^2 + 2\beta + 5)]\widetilde{\widetilde{i}}_4(\mathbb{C})\\
&+ 2(1+\beta)^2(m_u\langle\bar{u}u\rangle + m_d\langle\bar{d}d\rangle) i_2(\widetilde{\Psi}, 1)\Big\}\\
&+ \frac{1}{216 M^2} m_V f_V^{\parallel}(m_u\langle\bar{u}u\rangle + m_d\langle\bar{d}d\rangle)(5\beta^2 + 6\beta + 5)\Big[m_0^2\phi_V^{\parallel}(u_0) - 3m_V^2 i_2(\widetilde{\Phi}, 1)\Big]\\
&- \frac{1}{36 M^2} m_V^4 f_V^{\perp}(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle)[(1-\beta^2) i_1(\mathcal{T}, 1) - i_1(\mathcal{T}_4, 1)]\\
&- \frac{1}{576\pi^2 M^2} m_V^2 f_V^{\perp}(1-\beta^2)[3\langle g_s^2 G^2\rangle(m_u + m_d) - 40 m_0^2\pi^2(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle)]\psi_{3;V}^{\parallel}(u_0)\\
&- \frac{1}{96\pi^2} m_V^3 M^2 f_V^{\parallel}\Big\{(3\beta^2 + 2\beta + 3)[i_2(\mathcal{A}, 1) - 4i_2(\Phi, 1-2v)] + 2(5\beta^2 + 6\beta + 5) i_2(\widetilde{\Phi}, 1)\\
&- 2(3\beta^2 + 2\beta + 3) i_2(\Psi, 1-2v) - 4(1+\beta)^2 i_2(\widetilde{\Psi}, 1) - (3\beta^2 + 2\beta + 3) i_2(\mathcal{V}, 1-2v)\Big\}\\
&+ \frac{1}{128\pi^2} m_V^2 M^2\Big\{f_V^{\parallel} m_V[(3\beta^2 + 2\beta + 3)\mathbb{A}(u_0) + 4(5\beta^2 + 2\beta + 5)\widetilde{\widetilde{i}}_4(\mathbb{C})]
\end{aligned}
$$

$$
\begin{aligned}
&+ 12(m_u+m_d)(1-\beta^2) f_V^\perp \psi_{3;V}^\parallel(u_0)\Big\} \\
&- \frac{1}{24}\Big[m_V f_V^\parallel(3\beta^2+2\beta+3)(m_u\langle\bar{u}u\rangle + m_d\langle\bar{d}d\rangle)\phi_V^\parallel(u_0) + 6m_V^2 f_V^\perp(1-\beta^2)(\langle\bar{u}u\rangle+\langle\bar{d}d\rangle)\psi_{3;V}^\parallel(u_0)\Big] \\
&- \frac{1}{96\pi^2} m_V^4 f_V^\perp (m_u+m_d)(1-\beta^2) i_1(7\mathcal{T} - 8\mathcal{T}_3 - 2\mathcal{T}_4, 1) \\
&- \frac{1}{12\pi^2} m_V^5 f_V^\parallel (3\beta^2+2\beta+3)\Big[i_0(\Psi, 1-2v) - i_0(\widetilde{\Psi}, 1)\Big] \ , \qquad \text{(A.2)}
\end{aligned}
$$

$$
\begin{aligned}
&e^{m_V^2/4M^2}(\Pi_3^{f_1})^{asym}(u,d,s) = -\frac{1}{768\sqrt{2}M^2\pi^2} f_V^\perp \langle g_s^2 G^2\rangle m_V^2 (m_d - m_s)(1-\beta)(3+\beta) \\
&\times \left(E_\gamma - \ln\frac{M^2}{\Lambda^2}\right)[2\widetilde{i}_4(\mathbb{B}_T) + \widetilde{i}_4(\mathbb{C}_T)] \\
&- \frac{1}{72\sqrt{2}M^6} f_V^\parallel m_0^2 m_V^5 (m_d\langle\bar{s}s\rangle - m_s\langle\bar{d}d\rangle)(1-\beta)(3+\beta) i_0(\Psi, 1) \\
&- \frac{1}{2304\sqrt{2}M^4\pi^2} f_V^\perp m_V^4 (m_d - m_s)\langle g_s^2 G^2\rangle (1-\beta)(3+\beta) i_1(\mathcal{T} - 2\mathcal{T}_4, 1-2v) \\
&- \frac{1}{18\sqrt{2}M^4} f_V^\parallel m_V^5 (m_d\langle\bar{s}s\rangle - m_s\langle\bar{d}d\rangle)(1-\beta)(3+\beta) i_0(\Psi, 1) \\
&+ \frac{1}{3456\sqrt{2}\pi^2 M^2} m_V^2 f_V^\perp (1-\beta)[3\langle g_s^2 G^2\rangle (m_d - m_s)(3+\beta) \\
&- 16 m_0^2 \pi^2 (\langle\bar{d}d\rangle - \langle\bar{s}s\rangle)(5+2\beta)][2\widetilde{i}_4(\mathbb{B}_T) + \widetilde{i}_4(\mathbb{C}_T)] \\
&- \frac{1}{72\sqrt{2}M^2} m_V^4 f_V^\perp (\langle\bar{d}d\rangle - \langle\bar{s}s\rangle)(1-\beta)(3+beta) i_1(\mathcal{T} - 2\mathcal{T}_4, 1-2v) \\
&+ \frac{1}{48\sqrt{2}M^2} f_V^\parallel m_V^3 (m_d\langle\bar{d}d\rangle - m_s\langle\bar{s}s\rangle)(1+\beta)^2 \Big\{2[i_2(\Phi,1) - i_2(\widetilde{\Phi}, 1-2v)] - [i_2(\mathcal{A}, 1-2v) - i_2(\mathcal{V}, 1)]\Big\} \\
&- \frac{1}{64\sqrt{2}\pi^2} m_V^2 M^2 (m_d - m_s) f_V^\perp (1-\beta)(3+\beta)[2\widetilde{i}_4(\mathbb{B}_T) + \widetilde{i}_4(\mathbb{C}_T)] \\
&+ \frac{1}{24\sqrt{2}} m_V^2 f_V^\perp (\langle\bar{d}d\rangle - \langle\bar{s}s\rangle)(1-\beta)(3+\beta)[2\widetilde{i}_4(\mathbb{B}_T) + \widetilde{i}_4(\mathbb{C}_T)] \\
&+ \frac{1}{96\sqrt{2}\pi^2} m_V^4 f_V^\perp (m_d - m_s)(1-\beta)(3+\beta) i_1(\mathcal{T} - 2\mathcal{T}_4, 1-2v) \ . \qquad \text{(A.3)}
\end{aligned}
$$

Magnetic Type Coupling

The magnetic type coupling is determined by the coefficient of the structure $(\varepsilon \cdot p)\ \not{p}$:

$$
\begin{aligned}
&e^{m_V^2/4M^2}\Pi_1^{f_1+f_2}(u,d,s) = \frac{1}{64\pi^2} M^4 (1-\beta) f_V^\perp [m_d - m_s - (m_d + m_s)\beta]\phi_V^\perp(u_0) \\
&+ \frac{1}{192\pi^2} M^4 m_V f_V^\parallel \Big\{ i_3(\mathcal{V}, 1) + 9\psi_{3;V}^\perp(u_0) - \beta\Big[2 i_3(\mathcal{A}, 1-2v) \\
&- \beta i_3(\mathcal{V}, 1) - 3(2+3\beta)\psi_{3;V}^\perp(u_0)\Big]\Big\} \\
&+ \frac{1}{768\pi^2}\Big\{(1-\beta) f_V^\perp \left(\gamma_E - \ln\frac{M^2}{\Lambda^2}\right)\Big(8M^2[m_d(1-\beta) + m_s(1+\beta)] m_V^2 i_2(\mathcal{S}, 1)
\end{aligned}
$$

$$
\begin{aligned}
&+ [m_d(1-\beta) - m_s(1+\beta)](8M^2 m_V^2 i_2(\widetilde{\mathcal{S}},1) + \langle g_s^2 G^2\rangle \phi_V^\perp(u_0))\Big)\Big\} \\
&+ \frac{1}{384M^6} m_V m_0^2 f_V^\parallel \langle g_s^2 G^2\rangle (1-\beta^2)(m_s\langle \bar{d}d\rangle + m_d\langle \bar{s}s\rangle)\psi_{3;V}^\perp(u_0) \\
&+ \frac{1}{192M^4} m_V f_V^\parallel \langle g_s^2 G^2\rangle (1-\beta^2)(m_s\langle \bar{s}s\rangle + m_d\langle \bar{d}d\rangle)\psi_{3;V}^\perp(u_0) \\
&- \frac{1}{9216M^2\pi^2} m_V^2 f_V^\perp \langle g_s^2 G^2\rangle (1-\beta)\Big\{4[m_d(1-\beta) + 3m_s(1+\beta)]i_2(\mathcal{S},1) \\
&+ [m_d(1-\beta) - m_s(1+\beta)][4i_2(\mathcal{T}_1 - \mathcal{T}_2 + \mathcal{T}_3 - \mathcal{T}_4, 1-2v) - 3\mathbb{A}_T(u_0)]\Big\} \\
&+ \frac{1}{144M^2} m_V f_V^\parallel \Big\{m_V^2[m_d\langle \bar{d}d\rangle(1+\beta)^2 - m_s\langle \bar{s}s\rangle(\beta^2+6\beta+1)]i_2(\mathcal{A},1-2v) \\
&- m_V^2[m_d\langle \bar{d}d\rangle(1+\beta)^2 - m_s\langle \bar{s}s\rangle(3\beta^2+2\beta+3)]i_2(\mathcal{V},1) \\
&+ 3m_0^2(1-\beta^2)(m_d\langle \bar{s}s\rangle + m_s\langle \bar{d}d\rangle)\psi_{3;V}^\perp(u_0)\Big\} \\
&+ \frac{1}{384\pi^2} M^2 m_V^2 f_V^\perp (1-\beta)\Big\{[m_d(1-\beta) - m_s(1+\beta)]\Big[4i_2(\mathcal{T}_1 - \mathcal{T}_2 + \mathcal{T}_3 - \mathcal{T}_4, 1-2v) \\
&+ 4i_2(\widetilde{\mathcal{S}},1) - 3\mathbb{A}_T(u_0)\Big] + 8[m_d(1-\beta) + 2m_s(1+\beta)]i_2(\mathcal{S},1)\Big\} \\
&+ \frac{1}{96\pi^2} M^2 m_V^3 f_V^\parallel \Big[i_2(\mathcal{V},1) - 2\beta i_2(\mathcal{A},1-2v) + \beta^2 i_2(\mathcal{V},1)\Big] \\
&- \frac{1}{288} f_V^\perp (1-\beta)[\langle \bar{d}d\rangle(1-\beta) - \langle \bar{s}s\rangle(1+\beta)]\Big[4m_V^2 i_2(\mathcal{T}_1 - \mathcal{T}_2 + \mathcal{T}_3 - \mathcal{T}_4, 1-2v) \\
&- 3m_V^2\mathbb{A}_T(u_0) + 12M^2\phi_V^\perp(u_0)\Big] \\
&- \frac{1}{72} m_V^2 f_V^\perp (1-\beta)[\langle \bar{d}d\rangle(1-\beta) + 3\langle \bar{s}s\rangle(1+\beta)]i_2(\mathcal{S},1) \\
&- \frac{1}{1152\pi^2} f_V^\perp \langle g_s^2 G^2\rangle (1-\beta)[m_d(1-\beta) - m_s(1+\beta)]\phi_V^\perp(u_0) \\
&+ \frac{1}{432} m_0^2 f_V^\perp (1-\beta)[3\langle \bar{d}d\rangle(1-\beta) - 2\langle \bar{s}s\rangle(1+\beta)]\phi_V^\perp(u_0) \\
&+ \frac{1}{288} m_V f_V^\parallel \Big\{[m_d\langle \bar{d}d\rangle(1+\beta)^2 - m_s\langle \bar{s}s\rangle(\beta^2+6\beta+1)]i_3(\mathcal{A},1-2v) \\
&- [m_d\langle \bar{d}d\rangle(1+\beta)^2 - m_s\langle \bar{s}s\rangle(3\beta^2+2\beta+3)]i_3(\mathcal{V},1)\Big\} \\
&+ \frac{1}{48} m_V f_V^\parallel \Big\{\langle \bar{d}d\rangle[6m_s(1-\beta^2) + m_d(3\beta^2+2\beta+3)] \\
&+ \langle \bar{s}s\rangle[6m_d(1-\beta^2) + m_s(3\beta^2+2\beta+3)]\Big\}\psi_{3;V}^\perp(u_0)\ ,
\end{aligned}
\tag{A.4}
$$

$$
\begin{aligned}
&\quad e^{m_V^2/4M^2}\Pi_2^{f_1+f_2}(u,d,s) = \frac{1}{64\pi^2} M^4(1-\beta^2) f_V^\perp (m_u+m_d)\phi_V^\perp(u_0) \\
&- \frac{1}{192\pi^2} M^4 m_V f_V^\parallel \Big[(\beta^2+6\beta+1)i_3(\mathcal{A},1-2v) - (3\beta^2+2\beta+3)i_3(\mathcal{V},1) \\
&+ 3(1+\beta)^2\psi_{3;V}^\perp(u_0)\Big] \\
&- \frac{1}{768\pi^2}(m_u+m_d) f_V^\perp (1-\beta^2)\left(\gamma_E - \ln\frac{M^2}{\Lambda^2}\right)\Big[8M^2 m_V^2 i_2(\mathcal{S}-\widetilde{\mathcal{S}},1) - \langle g_s^2 G^2\rangle \phi_V^\perp(u_0)\Big]
\end{aligned}
$$

$$
\begin{aligned}
&+\ \frac{1}{1152M^6} m_V f_V^{\parallel} \langle g_s^2 G^2\rangle (1-\beta)^2 (m_u\langle\bar{d}d\rangle + m_d\langle\bar{u}u\rangle)(m_0^2+2M^2)\psi_{3;V}^{\perp}(u_0) \\
&+\ \frac{1}{9216M^2\pi^2} m_V^2 f_V^{\perp}\langle g_s^2G^2\rangle (1-\beta^2)(m_u+m_d)\Big[3\mathbb{A}_T(u_0) - 12 i_2(\mathcal{S},1) \\
&-\ 4 i_2(\mathcal{T}_1-\mathcal{T}_2,+\mathcal{T}_3-\mathcal{T}_4,1-2v)\Big] \\
&-\ \frac{1}{144M^2} m_V^3 f_V^{\parallel}(m_u\langle\bar{u}u\rangle + m_d\langle\bar{d}d\rangle)\Big[(\beta^2+6\beta+1) i_2(\mathcal{A},1-2v) - (3\beta^2+2\beta+3) i_2(\mathcal{V},1)\Big] \\
&+\ \frac{1}{384\pi^2} M^2 m_V^2 f_V^{\perp}(1-\beta^2)(m_u+m_d)\Big[4 i_2(\mathcal{T}_1-\mathcal{T}_2+\mathcal{T}_3-\mathcal{T}_4,1-2v) \\
&+\ 4 i_2(2\mathcal{S}+\widetilde{\mathcal{S}},1) - 3\mathbb{A}_T(u_0)\Big] \\
&+\ \frac{1}{96\pi^2} M^2 m_V^3 f_V^{\parallel}\Big[(3\beta^2+2\beta+3) i_2(\mathcal{V},1) - (\beta^2+6\beta+1) i_2(\mathcal{A},1-2v)\Big] \\
&-\ \frac{1}{72} f_V^{\perp}(1-\beta^2)(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle)\Big\{m_V^2\Big[i_2(\mathcal{T}_1-\mathcal{T}_2+\mathcal{T}_3-\mathcal{T}_4,1-2v) \\
&+\ 3 i_2(\mathcal{S},1)\Big] + 3M^2\phi_V^{\perp}(u_0)\Big\} \\
&+\ \frac{1}{864} f_V^{\perp}(1-\beta^2)(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle)\Big[10 m_0^2\phi_V^{\perp}(u_0) + 9 m_V^2\mathbb{A}_T(u_0)\Big] \\
&-\ \frac{1}{1152\pi^2} f_V^{\perp}\langle g_s^2G^2\rangle(1-\beta^2)(m_u+m_d)\phi_V^{\perp}(u_0) \\
&-\ \frac{1}{288} m_V f_V^{\parallel}(m_u\langle\bar{u}u\rangle + m_d\langle\bar{d}d\rangle)\Big[(\beta^2+6\beta+1) i_3(\mathcal{A},1-2v) - (3\beta^2+2\beta+3) i_3(\mathcal{V},1)\Big] \\
&-\ \frac{1}{48} m_V f_V^{\parallel}\Big\{\langle\bar{u}u\rangle[m_u(1+\beta)^2 - 2m_d(1-\beta)^2] - \langle\bar{d}d\rangle[2m_u(1-\beta)^2 - m_d(1+\beta)^2]\Big\}\psi_{3;V}^{\perp}(u_0)\,. \qquad \text{(A.5)}
\end{aligned}
$$

$$
\begin{aligned}
&e^{m_V^2/4M^2}(\Pi_3^{f_1+f_2})^{asym}(u,d,s) = \frac{1}{96\sqrt{2}\pi^2}(1-\beta) f_V^{\perp} m_V^2 M^2 (m_d-m_s)\left(E_\gamma - \ln\frac{M^2}{\Lambda^2}\right) \\
&\times\ [4(1+\beta) i_2(\mathcal{T}_1-\mathcal{T}_2,1) + (3+\beta) i_2(\mathcal{T}_3-\mathcal{T}_4,1)] \\
&-\ \frac{1}{2304\sqrt{2}\pi^2M^2} m_V^2(m_d-m_s) f_V^{\perp}\langle g_s^2G^2\rangle(1-\beta)[(1-\beta) i_2(\widetilde{\mathcal{S}},1-2v) + (1+3\beta) i_2(\mathcal{T}_1-\mathcal{T}_2,1)] \\
&+\ \frac{1}{24\sqrt{2}M^2} m_V(m_d\langle\bar{d}d\rangle - m_s\langle\bar{s}s\rangle) u_0 f_V^{\parallel} q^2 (1+\beta)^2[i_2(\mathcal{A},1) - i_2(\mathcal{V},1-2v)] \\
&+\ \frac{1}{96\sqrt{2}\pi^2} m_V^2(m_d-m_s) M^2 f_V^{\perp}(1-\beta)[(1-\beta) i_2(\widetilde{\mathcal{S}},1-2v) + (5+7\beta) i_2(\mathcal{T}_1-\mathcal{T}_2,1) \\
&+\ (3+\beta) i_2(\mathcal{T}_3-\mathcal{T}_4,1)] \\
&+\ \frac{1}{96\sqrt{2}} m_V(m_d\langle\bar{d}d\rangle - m_s\langle\bar{s}s\rangle) f_V^{\parallel}(1+\beta)^2[i_3(\mathcal{A},1) - i_3(\mathcal{V},1-2v) \\
&-\ \frac{1}{72\sqrt{2}} m_V^2(\langle\bar{d}d\rangle - \langle\bar{s}s\rangle) f_V^{\perp}(1-\beta)[(1-\beta) i_2(\widetilde{\mathcal{S}},1-2v) + (1+3\beta) i_2(\mathcal{T}_1-\mathcal{T}_2,1)]\,. \qquad \text{(A.6)}
\end{aligned}
$$

The functions i_n, $\widetilde{i}_4$ and $\widetilde{\widetilde{i}}_4$ are defined as

$$
\begin{aligned}
i_0(\phi, f(v)) &= \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v)(k-u_0)\theta(k-u_0) \ , \\
i_1(\phi, f(v)) &= \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v)\theta(k-u_0) \ , \\
i_2(\phi, f(v)) &= \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v)\delta(k-u_0) \ , \\
i_3(\phi, f(v)) &= \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v)\delta'(k-u_0) \ , \\
\widetilde{i}_4(f(u)) &= \int_{u_0}^1 du f(u) \ , \\
\widetilde{\widetilde{i}}_4(f(u)) &= \int_{u_0}^1 du (u-u_0) f(u) \ ,
\end{aligned}
$$

where

$$
k = \alpha_q + \alpha_g \bar{v} \ , \qquad u_0 = \frac{M_1^2}{M_1^2 + M_2^2} \ , \qquad M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2} \ .
$$

In the expressions for $\Pi_i^{(\alpha)}$, the contributions of the higher states and continuum are subtracted using the replacements:

$$
\begin{aligned}
e^{-m_V^2/4M^2} M^2 \left(\ln \frac{M^2}{\Lambda^2} - \gamma_E \right) &\rightarrow \quad \int_{m_V^2/4}^{s_0} ds e^{-s/M^2} \ln \frac{s - m_V^2/4}{\Lambda^2} \\
e^{-m_V^2/4M^2} \left(\ln \frac{M^2}{\Lambda^2} - \gamma_E \right) &\rightarrow \quad \ln \frac{s_0 - m_V^2/4}{\Lambda^2} e^{-s_0/M^2} + \frac{1}{M^2} \int_{m_V^2/4}^{s_0} ds e^{-s/M^2} \ln \frac{s - m_V^2/4}{\Lambda^2} \\
e^{-m_V^2/4M^2} \frac{1}{M^2} \left(\ln \frac{M^2}{\Lambda^2} - \gamma_E \right) &\rightarrow \quad \frac{1}{M^2} \ln \frac{s_0 - m_V^2/4}{\Lambda^2} e^{-s_0/M^2} + \frac{1}{s_0 - m_V^2/4} e^{-s_0/M^2} \\
&\qquad + \frac{1}{M^4} \int_{m_V^2/4}^{s_0} ds e^{-s/M^2} \ln \frac{s - m_V^2/4}{\Lambda^2} \\
e^{-m_V^2/4M^2} M^{2n} &\rightarrow \quad \frac{1}{\Gamma(n)} \int_{m_V^2/4}^{s_0} ds e^{-s/M^2} \left(s - m_V^2/4 \right)^{n-1}
\end{aligned}
$$

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Figure captions

Fig. (1) The dependence of the electric coupling constant f_1 of $p \to p\rho^0$ transition on Borel mass M^2 for the three fixed values of the parameter β: $\beta = -1, \pm 5$, and two fixed values of the vacuum threshold s_0: $s_0 = 2.25\ GeV^2$ and $s_0 = 2.75\ GeV^2$

Fig. (2) The dependence of the electric coupling constant f_1 of $p \to p\rho^0$ transition on $\cos\theta$ for the two fixed values of the vacuum threshold s_0: $s_0 = 2.25\ GeV^2$ and $s_0 = 2.75\ GeV^2$, and for the Borel mass at $M^2 = 1\ GeV^2$.

Fig. (3) The same as in Fig. (1), but for the coupling constant $(f_1 + f_2)$.

Fig. (4) The same as in Fig. (2), but for the coupling constant $(f_1 + f_2)$.

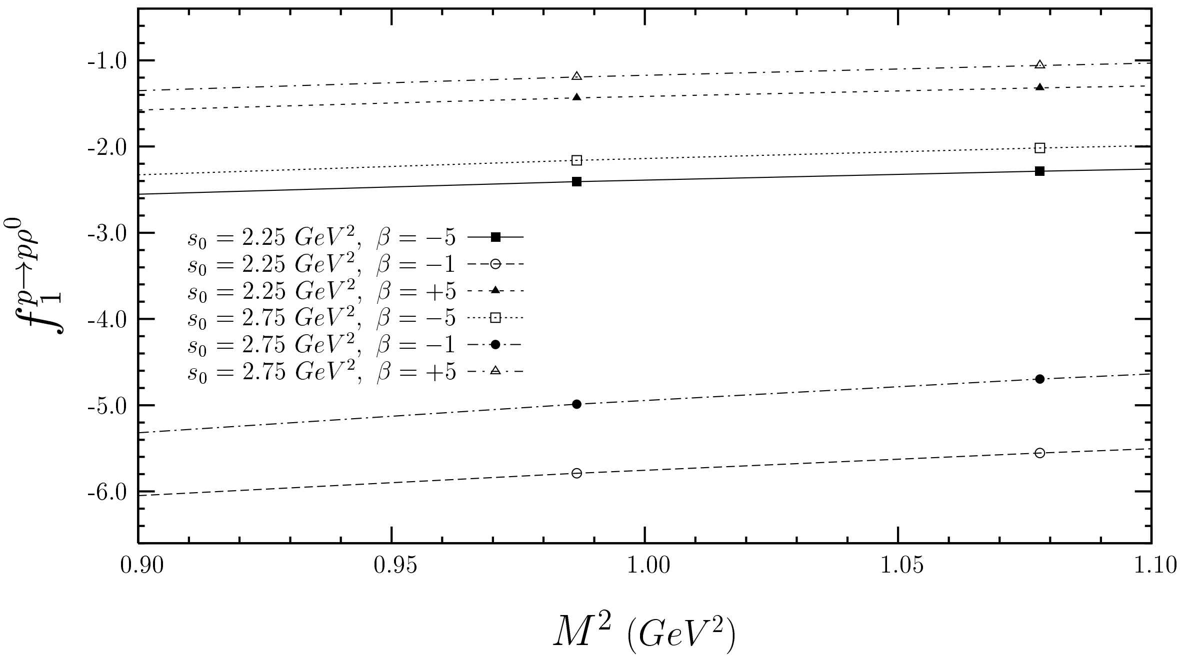

Figure 1:

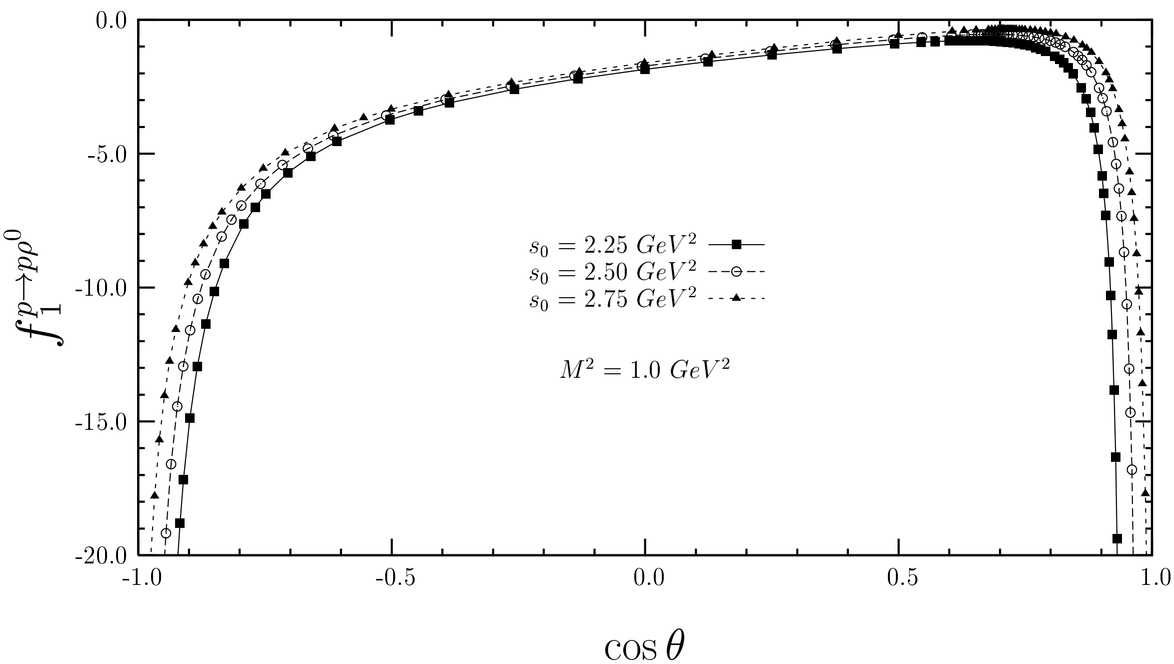

Figure 2:

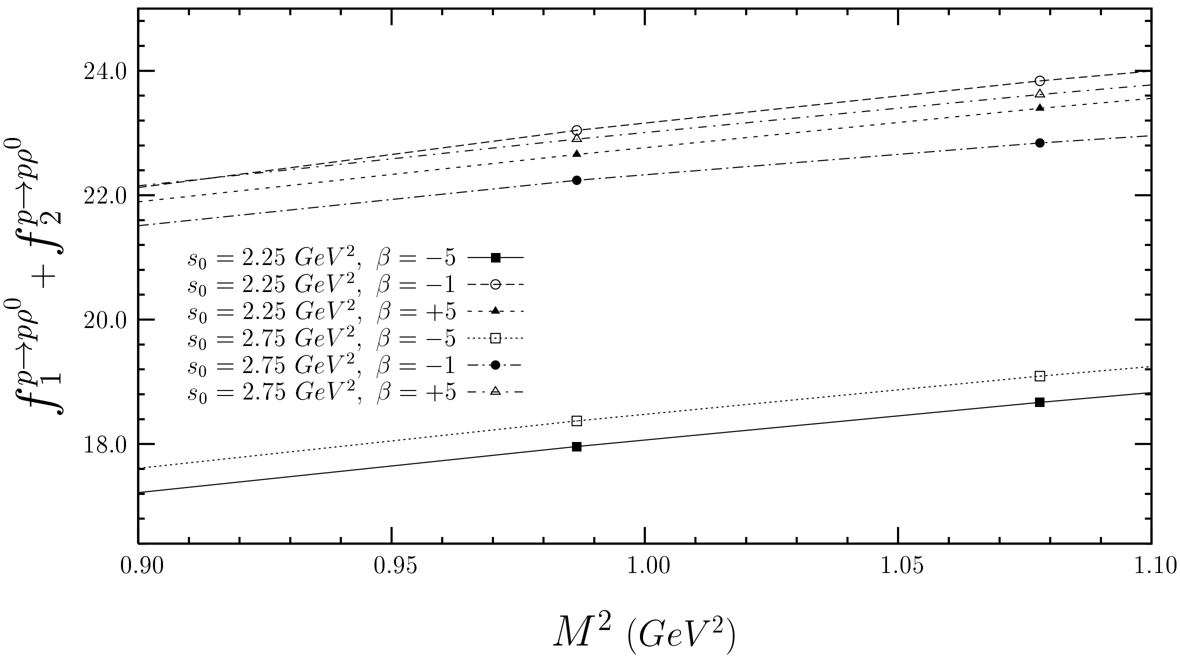

Figure 3:

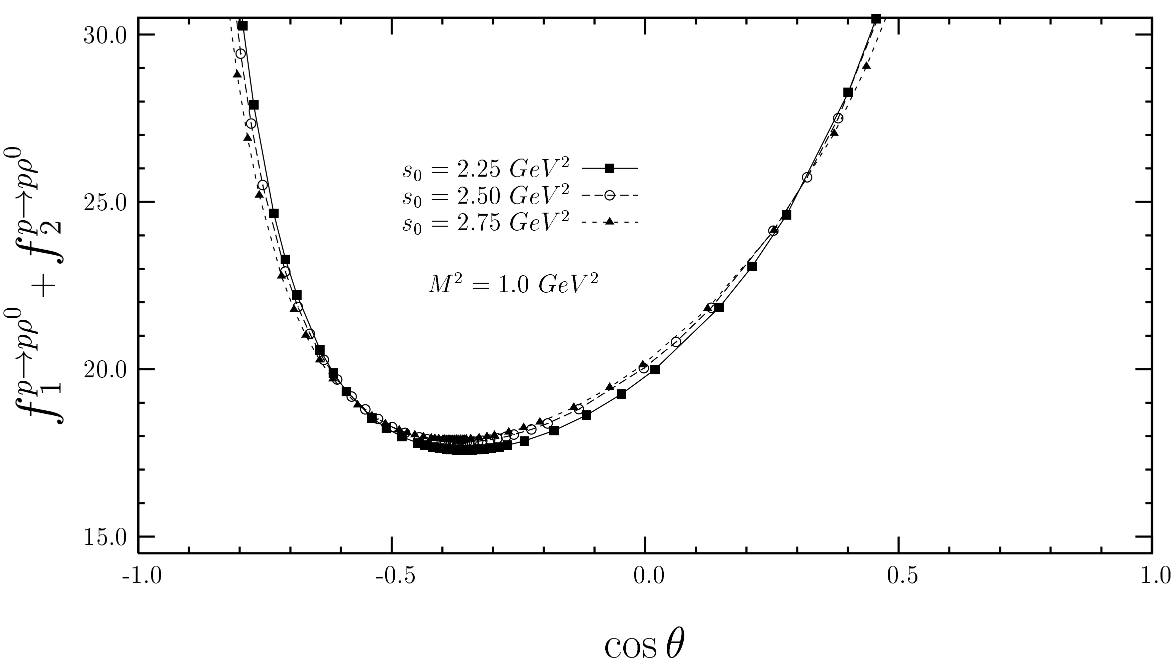

Figure 4: